

## Homework 3 Answer key

### Problem 1

(a) **(+20 pts)** To find optimal effort we should consider Agent's optimization problem:

$$\max_e [Eu(w(e, \alpha, \beta))]$$

$$\text{FOC: } \frac{\partial Eu}{\partial e} = -\exp(-\alpha - 2\beta - \beta e + \frac{e^2}{4} + \frac{\beta^2}{2}) \times (-\beta + \frac{e}{2}) = 0$$

$$e^* = 2\beta$$

(b) **(+30 pts)** To find optimal contract the principal's problem should be considered:

General form:

$$\max_{\alpha, \beta} E[x - w(e^*(\alpha, \beta), \alpha, \beta)]$$

s.t.

$$IR: Eu[w(e^*(\alpha^*, \beta^*))] \geq u_0$$

$$IC: \frac{\partial Eu[w(e^*(\alpha^*, \beta^*))]}{\partial e} = 0$$

Both conditions hold with equality. Solutions for IC ( $e^* = 2\beta$ ) was obtained in (a)

Maximization problem turns into:  $\max_{\beta} [2 + 2\beta - (-2\beta - 0,5\beta^2) - \beta(2 + 2\beta)]$

$$\beta^* = \frac{2}{3} \alpha^* = -\frac{14}{9} \quad (e^* = \frac{4}{3})$$

(c) **(+15 pts)**

$$E(w^*) = \frac{2}{3}$$

$$Var(w^*) = \frac{4}{9}$$

$$Eu(\alpha^*, \beta^*, e^*) = -1$$

(d) **(+10 pts)** An answer depends on the nature of the production shock. If it involves a significant part of common shock, relative contract should be applied, so common shock could be filtered out (better for risk-averse agents) and lower average wage could be paid (benefits the risk neutral principal). If only specific shock is assumed, a piece rate should be applied.

### Problem 2 (+25 pts)

The weakest equilibrium we can find here is a NE. If any team wins it qualifies with Italy and the loser wants to deviate. So, NE will be in the draw.

- For a draw 0-0 or 1-1, Denmark qualifies, so Sweden wants to deviate.
- For any draw 2-2 and above, both teams qualify and nobody wants to deviate.

However, the situation is a bit different that in our usual games. In order to reach any NE from 3-3 and above, one of teams has fist to deviate from the NE 2-2 (that is score a third goal). Scoring a third goal at the 2-2 requires effort and risk. If teams make it to 2-2 they will not want to deviate from there. As long as the score is below 2-2 at least one team will have to put the effort and the risk to score and deviate.