a. Firms solve $\begin{cases}\pi_1 = (a - bp_1 + dp_2)p_1 \rightarrow \max \quad p_1 \\ \pi_2 = (a - bp_2 + dp_1)p_2 \rightarrow \max \quad p_2\end{cases}$, which results in

$$p_1^* = p_2^* = \frac{a}{2b - d} \; and \; \pi_1^* = \pi_2^* = \frac{ba^2}{(2b-d)^2}.$$

b. Since $\frac{\partial \pi_i^*}{\partial \mathrm{d}} > 0$, firms prefer higher d ie the sensitivity of demand to the rival's price.

c. The cartel maximises $\pi_1 + \pi_2 = (a - bp_1 + dp_2)p_1 + (a - bp_2 + dp_1)p_2 \; w.r.t. \, p_1 and \; p_2$ The solution

is $p_1^{**} = p_2^{**} = \frac{\mathrm{a}}{2(\mathrm{b}-\mathrm{d})}$ and the profits are $\pi_1^{**} = \pi_2^{**} = \frac{\mathrm{a}^2}{4(\mathrm{b}-\mathrm{d})}$.

d. Firms i's profit is $\pi_1 = \left(a - bp_i + d\frac{(\sum_{j \neq i} p_j)}{N-1}\right)p_i$. Maximizing this with respect to p_i for all firms simultaneously leads to $p_i^* = p_j^* = \frac{a}{2b-d}$, the same result as in (a).

e. The prices do not change but the market size is larger as the total trading volume is Nq_i^* is the same as in (a) As a result, consumer surplus increases proportionally with the number of firms.