

Homework 3

Due October 17, 2017

Homework will be collected at the end of the lecture on the day it is due. Submissions in any other time or manner will be ignored. The maximum score is 100. Unprofessionally looking papers or unnamed or unstapled sheets or improperly labelled questions or bad handwriting will result to a penalty up to 50% at the discretion of the grader. Plagiarism will be prosecuted and perpetrators will be in very deep trouble.

1. A risk neutral and residual claimant principal considers offering the contract $w = \alpha + \beta x$ to an agent, who produces output according to $x = 2 + e + \epsilon$ where e is the (unobservable by the principal) effort the agent exerts and $\epsilon \sim N(0,1)$ is a production shock. The agent's utility function is:

$$u(w, e) = -\exp\left(-w + \frac{e^2}{4}\right).$$

The reservation utility for the agent is -1. Use the standard property for the lognormal distribution that: $x \sim N(\mu, \sigma^2) \Rightarrow E[\exp(x)] = \exp[\mu + \sigma^2/2]$ to derive the following.

- (a) The optimal e . [20p]
 - (b) The optimal contract (α^*, β^*) . [30p]
 - (c) The expected wage, the variance of the expected wage and the expected utility for the agent [15p]
 - (d) If the agent found another agent identical to the agent above, would she want to offer him the same contract you derived in (b) or she should offer both agents a relative contract? Explain. [10p]
2. The last qualifying match of Group C in the UEFA 2004 Euro was Sweden vs. Denmark. With all other matches already played the standings were

<i>Team</i>	<i>Points</i>	<i>Goals</i>
Italy	5	2
Denmark	4	1
Sweden	4	1
Bulgaria	0	0

In every soccer match, a win is awarded 3 points, a draw 1 point per team and a loss 0 points. The two teams with the most points in the group would qualify to the next round but teams were indifferent if they would qualify first or second. In case more than two teams ended up at the top with the same number of points, the number of goals scored would be used to break the tie. If the number of goals is also equal, Denmark and Italy would qualify over Sweden (because of further tie-breaking rules). The match was scheduled for June 22, 2004. Using terms we have covered in the course explain why the most likely result for this game would be 2-2. [This is a real case – number of goals is slightly modified to simplify tie-breaking but does not alter the facts]. [25p]