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# Lecture 4

Contracts

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Industrial
Economics

## Before we begin

- Saturday's ***labs***
- Homework 2 available, due next Tuesday
- Clarity of homework
- Examples
- Theory of the firm, easy?



## Economic rationality

- On February 17th 2001 notorious Greek robber ***Kostantinos Passaris*** is transferred from prison to hospital for scheduled exams
- During the wait someone passes to the criminal a loaded ***9mm pistol***
- Within a moment Passaris ***kills*** two of the guards and ***seriously injures*** the third one
- While fleeing the scene, he comes ***face to face*** with the driver of the police van who was waiting at the yard

  *"I will not kill you because my last bullet is worth more than your […] life"*





## Is Johnny rational?

*Johnny visits a diner and asks for coffee and dessert. The waitress tells him that there is only **apple pie** and **banana putting**. Johnny chooses the apple pie*

*After a few minutes, the waitress comes back and informs Johnny that there is also **carrot cake**, in case he is interested to change his order*

*Johnny thinks for a moment and says:*

*-Yes, I want to change… bring me the banana putting!*





## Von Neumann – Morgenstern rationality

Consider 3 lotteries $L, M, N$

1. ***Completeness***

   $L, M, N$ can be ranked using the relationships $\prec, \preccurlyeq, =$
2. ***Transitivity***

   $L \preccurlyeq M$ and $M \preccurlyeq N \Rightarrow L \preccurlyeq N$ (preferences are consistent)
3. ***Continuity***

   $L \preccurlyeq M \preccurlyeq N \Rightarrow \exists\, p \in [0,1] \ : \ pL + (1-p)N = M$
   (you can always mix a middle option)
4. ***Independence***

   $L \prec M \Rightarrow pL + (1-p)N \prec pM + (1-p)N, \forall N \ \& \ \forall\, p\ (0,1]$
   (same mixtures of irrelevant lotteries cancel out)





## Rational preferences

- Preferences are ***rational*** when they obey the VNM axioms
- The Von Neumann–Morgenstern ***theorem***

  *for any VNM-rational agent, there exists function $u$ mapping any lottery to a real number such that*

  $$\forall\, M \prec N \Rightarrow u(M) < u(N)$$

## The profit maximization hypothesis

- A ***common assumption*** in economic theory is that firms maximize (expected) profits
  - This is probably what the ***owners*** would like to do
  - But, most firms are ***not run*** by the owners
- Managers are likely to have ***other objectives*** than profit maximization
- How ***tangible*** is profit maximization?



## Deviation of interests

- There exists an ***asymmetry*** problem
- Managers:
  - ***Know more*** about how to run the business
  - Have more ***information*** for the operations
  - Have the opportunity to take ***hidden action***
  - May incur ***different costs*** than owners
  - May avoid, fool or corrupt ***monitoring***
- Questions:
  - Can we ***align*** the interest of the principal and the agent?
  - What ***mechanisms*** are there to limit managerial discretion?



## A discrete contract

- An risk neutral owner (principal - she) hires a risk averse manager (agent - he) for a ***salary*** $w$
- The agent chooses his ***effort:*** $e \in \{0, e^*\}$
- Agent's ***utility*** is:

  $$u(w-e), \quad \partial u/\partial w > 0, \quad \partial^2 u/\partial w^2 < 0$$

  that is, effort causes $e$ units of ***monetary disutility***
- Agent's ***reservation*** wage is: $w_o : u_o \equiv u(w_o)$
- Principal is risk neutral and receives ***gross profit***

  $\pi \in \{\pi_L, \pi_H\}$, where $\pi_L < \pi_H$
- The realization of $\pi$ depends ***stochastically*** on the agent's effort choice, $e$





## Stochasticity

- To simplify the notation we ***define***

| | |
|---|---|
| $p_{H,e^*} \equiv p(\pi = \pi_H \mid e = e^*)$ | $p_{L,e^*} \equiv p(\pi = \pi_L \mid e = e^*)$ |
| $p_{H,0} \equiv p(\pi = \pi_H \mid e = 0)$ | $p_{L,0} \equiv p(\pi = \pi_L \mid e = 0)$ |

- For $\boldsymbol{e = e^*}$, $p_{H,e^*} + p_{L,e^*} = 1$
- For $\boldsymbol{e = 0}$, $p_{L,0} + p_{H,0} = 1$
- It must be $p_{H,e^*} > p_{H,0}$

  increasing effort ***must increase*** the probability of $\pi_H$
- Analogously $p_{L,0} > p_{L,e^*}$

  decreasing effort ***must increase*** the probability of $\pi_L$
- This property is known as ***stochastic dominance***



## Stochastic dominance

- Stochastic dominance does not imply that the ***most probable*** result under high effort would be success ($\pi_H$)

  that is, $p_{H,e^*} > p_{L,e^*}$ may ***not*** be necessarily true
- Imagine a ***very difficult*** project in which
  - $e = 0$ gives the project 1% chance to succeed
  - $e = e^*$ gives the project 2% chance to succeed
- The setting ***satisfies*** stochastic dominance: $\overset{2\%}{p_{H,e^*}} > \overset{1\%}{p_{H,0}}$
- But does ***not satisfy*** that $\overset{2\%}{p_{H,e^*}} > \overset{98\%}{p_{L,e^*}}$





## The contract

- The principal wants to come up with a ***compensation scheme*** to ***incentivize*** the agent to ***exert*** the amount of effort ***she wants***
- The principal can make $w$ ***contingent*** on some condition
- That is,

  $$w(e) = \begin{cases} w_1, & if\ e = 0 \\ w_2, & if\ e = e^* \end{cases}$$

  the condition may involve ***any variable*** that the principal can ***observe*** and ***verify (contractible)***
- The principal will ***offer take-it-or-leave-it*** the scheme to the agent ***before*** he chooses $e$

## Contractible effort

- The contract will be contingent on ***effort***
- Assuming that the principal wants to ***implement*** $e^*$, the scheme would simply be:

$$w(e) = \begin{cases} 0, & if\ e = 0 \\ w_o + e^*, & if\ e = e^* \end{cases}$$

- Assuming that the principal wants to ***implement zero effort***, the scheme would be:

$$w(e) = \begin{cases} w_o, & if\ e = 0 \\ 0, & if\ e = e^* \end{cases}$$

if she offers flat $w_o$ the agent will again select $e = 0$





## Unobservable effort

- The contract now ***cannot*** be contingent on effort
- It can be contingent on ***profit***

  $\pi$ ***stochastically*** depends on agent's ***costly*** choice, $e$
- The principal faces an ***informational asymmetry*** (?)

  ***- moral hazard!***
- Contract will ***look like***:

$$w(\pi) = \begin{cases} w_L, & if\ \pi = \pi_L \\ w_H, & if\ \pi = \pi_H \end{cases}$$

- The principal is ***risk neutral*** and ***maximizes*** $E(\pi - w)$

  must set $w(\pi)$, so that the agent will ***accept*** the offer and ***exert*** the effort required by the principal





## Incentive conditions

- In general, ***any*** incentive problem is governed by two basic ***conditions*** which should hold at equilibrium

  the ***effort level*** that the principal wants to ***implement***

1. Individual rationality (IR)

$$Eu(w, e^*) \geq u_o$$

that is, at $e^*$ the expected utility of the agent should be at least as good as the ***outside option***

2. Incentive compatibility (IC)

$$Eu(w, e^*) \geq Eu(w, e') \quad \forall\, e' \neq e^*$$

that is, at $e^*$ the expected utility of the agent should be at least as good as the expected utility ***of any other effort choice***





## IC and IR for $e = e^*$

- The principal must calculate the agent's ***expected utility*** for each possible effort level:
  - $EU_{e=0} = p_{L,0} \cdot u(w_L) + (1 - p_{L,0}) \cdot u(w_H)$
  - $EU_{e=e^*} = p_{H,e^*} \cdot u(w_H - e^*) + (1 - p_{H,e^*}) \cdot u(w_L - e^*)$
- The agent will ***sign the contract*** iff

$$EU_{e=e^*} \geq u_o \qquad \text{(IR)}$$

- The agent will ***exert*** $e^*$ iff

$$EU_{e=e^*} \geq EU_{e=0} \qquad \text{(IC)}$$

- The agent will sign the contract even if the IR and IC hold with ***equality*** – no reason for ***rents***





## The optimal scheme for $e = e^*$

- The principal's ***expected payoff*** is

$$E\Pi_{e=e^*} = p_{H,e^*} \cdot (\pi_H - w_H) + (1 - p_{H,e^*}) \cdot (\pi_L - w_L)$$

- The principal's ***problem*** becomes

$$\max_{w_L, w_H} E\Pi_{e=e^*}$$

s.t. $EU_{e=e^*} = u_o$ and $EU_{e=e^*} = EU_{e=0}$

- The combination $(w_H^*, w_L^*)$ which solves the principal's problem will ***satisfy***:

$$w_H^* > w_L^*$$

the agent is ***rewarded*** if profit turns out high, no matter the actual effort





## Comparison of $Ew$ when $e = e^*$

- Expected wage for the agent under unobservable effort:

$$Ew_{un} = p_{H,e^*} \cdot w_H^* + (1 - p_{H,e^*}) \cdot w_L^*$$

- Under observable effort:

$$Ew_{ob} = w_o + e^*$$

- Comparison

$$Ew_{un} > Ew_{ob}$$

  - Agent is ***risk averse*** – needs to receive ***risk premium*** for the case that effort is high but profit low
  - The principal's ***net profit*** will be lower

## Unobservable effort and $e = 0$

- The principal's problem:

$$\max_{w_L, w_H} E\Pi_{e=0}$$

s.t. $EU_{e=0} = u_o$ and $EU_{e=0} = EU_{e=e^*}$

- The IC is ***not binding*** in this case

  if the agent is offered wage for $e = 0$ but deviates to $e = e^*$ with the same wage, the principal does ***not*** mind

$$E\Pi_{e=e^*} > E\Pi_{e=0}$$

- The IR is still binding

  the principal will offer a flat $w_o$ to ***just ensure participation***

- ***Expected payoff*** is

$$E\Pi_{e=0} = p_{L,0} \cdot (\pi_L - w_o) + (1 - p_{L,0}) \cdot (\pi_H - w_o)$$



Thank you!

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