The Prisoners’ dilemma

The Prisoners’ dilemma is the most useful game in Industrial Organization

- Two suspects are accused of committing a crime
- They are both arrested and placed in separate cells
- Each has been asked to confess to the crime
- Confession by either suspect will make the work of the prosecutor easier, so, she is offering them a deal.

The offer of the prosecutor to each prisoner is

<table>
<thead>
<tr>
<th>Bonus</th>
<th>Clyde</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confess</td>
<td>-5, -5</td>
</tr>
<tr>
<td>Deny</td>
<td>-10, -1</td>
</tr>
</tbody>
</table>

The Prisoners’ dilemma – equilibrium

The NE is (C,C) and it is socially suboptimal

Both players would accept to collude by moving to (D,D) as long as there is commitment to do so

Players prefer to receive -5 rather than expose themselves to the danger of receiving -10.

Repeated games

In repeated games, two important things may happen:

1. Players can develop reputations
2. Players have a chance for retaliation

Players may jointly try to impose an outcome that is better than the NE (but not NE)

- There is still incentive for cheating in the static game
- In the repetition process, though, this incentive may be eliminated
- The cheater can get away with a higher payoff for that round
- \textbf{BUT}, starting from the next round, the player who was cheated upon will retaliate by choosing the NE strategy.

Tit-for-tat strategy

Retaliation is often called \textit{tit-for-tat strategy}

“I trust you and you play the collusive strategy but if you cheat, I will be playing the Nash strategy forever”

The tit-for-tat strategy is a trigger strategy

everyone trusts everyone else until someone pushes the trigger

Once the trigger is pushed

- The cheater loses from next period (social optimum → NE)
- The fair player (ones too but can’t trust the cheater anymore)
- For the fair player the NE is better than to be cheated upon

Can repetition lead to the collusive outcome in a prisoners’ dilemma type of game?

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Indefinite repetition

- What if the game is infinitely repeated or randomly terminated?
- Tit-for-tat strategy makes sense
  - If a player cheats
  - The other player will be playing the Nash strategy, forever
  - The cheater will get high payoff for a single period but from the next period will revert to the lower Nash payoff
- The threat of retaliation can be credible
  - If the one-time payoff from cheating does not exceed the NPV of the infinite stream of NE payoffs.

Sub-perfection

- If a trigger strategy is Nash for the repeated game starting at period 1, it will be also Nash for the game starting at any period
  - because those games are identical
- In this case, we say that the trigger is a sub-game-perfect NE (SPNE)
  - that is, a NE that survives in every sub-game that includes it.

Finite repetition

- What if the game is repeated a known finite number of times?
- Let’s take things from the end
  - In the last period there is no possibility of retaliation, thus, everyone will cheat
  - If everyone cheats in the last period, there is no fear of retaliation to the second to last period
  - So, there is no retaliation for any period
- The threat of retaliation is not credible
  - collusion is not sustainable.

Dynamic games of complete info

- Consider a game between an entrant and an incumbent:

<table>
<thead>
<tr>
<th></th>
<th>Enter</th>
<th>Pass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inc.</td>
<td>Accommodate</td>
<td>Fight</td>
</tr>
<tr>
<td></td>
<td>1,2</td>
<td>-1,-3</td>
</tr>
<tr>
<td></td>
<td>2,0</td>
<td>-3,0</td>
</tr>
</tbody>
</table>

- What happens if the entrant plays first?
- The incumbent can threat the entrant
  - if you ‘enter’ I will ‘fight’ – I may lose 1 but you will lose 3!
  - However, this threat will be empty
  - once the entrant has entered the incumbent will want to accommodate.

Credibility

- The entrant can understand from the beginning that the threat is empty
  - so, will decide to enter
- This threat has a credibility problem
  - if incumbent played first, things would be different
- Dynamic inconsistency: A strategy may be optimal ex ante but sub-optimal ex-post, depending on the evolution of the game
  - the problem here is that the NE concept cannot distinguish empty threats.

Sequential rationality

- The equilibrium concept in a dynamic game must satisfy sequential rationality
  - for each player, at every stage, strategy must be optimal from that node and on
- This seems easier if
  1. We represent the game in a tree form
  2. Split it in smaller sub-games from each node
  3. Then, analyze it using backward induction
- Zermelo’s theorem: In every finite game of perfect info backward induction can yield a NE in pure strategies – if payoffs are asymmetric the equilibrium will be unique.
Sub-game perfection

- A sub-game is a subset of the game that begins with a node and includes all consequent nodes and branches every sub-game is a game on its own
- SPNE: A profile of strategies that is a NE in every sub-game of this game that contains this profile

Intuition

- The last game is a standalone game
- As it results in a unique NE it may be replaced with its NE
- This logic is applied then to the next-to-last game

Dynamic games are usually represented with a succinct game tree

BMW vs. Benz: product choice problem

- BMW and Mercedes each wish to introduce a new type of vehicle in the market
  - Either a Compact Utility Vehicle (CUV)
  - Or a Compact Cabriolet (Cabrio)
- Firms will be better off if they introduce a different type of vehicle
  - Because the demands in those markets are small and cannot accommodate two competing sellers
  - Plus firms need to sell a high quantity to reach their MES
- Decisions are non-cooperative.

Product choice problem

- If BMW hears that Mercedes is introducing a CUV, its best action is to produce a Cabrio
- Bottom left corner is Nash equilibrium
- What is other Nash Equilibrium?

Sequential moves

- If both firms announce their decisions independently and simultaneously, they may both lose money
- What if Mercedes sped up production and introduced a new model first?
- Now there is a sequential game
- BMW will have to produce the opposite of what Mercedes produced

Extensive form

- The above bi-matrix does not depict the game with clarity anymore
- We have to represent possible moves in the extensive form of a decision tree
  - this allows to work backward from the best outcome for BMW

Decision tree
First-mover advantage

- In some games the first-mover (leader) has advantage in the previous example the company who would choose product first gained 12, while the follower only 10.
- The follower can threaten the leader, but an empty threat will not reverse the advantage.
- Let's now examine a strategy of preemptive restriction.

Energon vs. Orange

Orange (1st)

<table>
<thead>
<tr>
<th>Energon</th>
<th>Thin frame</th>
<th>Thick frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thin battery</td>
<td>3, 6</td>
<td>3, 0</td>
</tr>
<tr>
<td>Thick battery</td>
<td>1, 1</td>
<td>2, 3</td>
</tr>
</tbody>
</table>

- Orange Inc. produces cellphones.
- Energon produces batteries for cellphones.
- Sequential game with Orange as the first-mover.
- Orange does best by producing thin cellphones.
- Orange knows that Energon will then produce thin batteries.
- Energon prefers to make thick batteries.

Threat

Can Energon induce Orange to produce thick frames? recall that Energon moves after Orange.

Suppose Energon threatens to produce thick batteries regardless what Orange does not credible! once Orange has produced thin frames, Energon will lose if carries out its threat.

Can Energon make the threat credible?

A credible threat

Energon burns down (!) the production line of small batteries.

Energon now can credibly threaten that it will produce "big batteries".

Of course, Energon wants to make sure that Orange executives hear about the fire :).

Irrationality

- If a player gets the reputation of being "irrational" threats might be in fact credible.
  Irrational individuals do not always make profit maximizing decisions.
- In some occasions reputation of irrationality can lead to a significant advantage.
  - Opponents cannot estimate you with logic.
  - Your threats will be taken more seriously.

Thank you!
WARNING

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