Kosmas Marinakis, Ph.D.

Lecture 7

Dynamic games

Industrial Economics

The Prisoners' dilemma

- The Prisoners' dilemma is the ***most useful game*** in Industrial Organization
  - Two suspects are ***accused*** of committing a crime
  - They are both arrested and placed in ***separate*** cells
  - Each has been ***asked to confess*** to the crime
  - Confession by either suspect will make the work of the prosecutor easier, so, she is offering them a ***deal***

Offer

		Clyde: Confess	Clyde: Deny
Bonnie	Confess	-5, -5	-1, -10
Bonnie	Deny	-10, -1	-2, -2

- The ***offer*** of the prosecutor to each prisoner is
  - If you confess and your partner does not, you get 1 year and your partner 10
  - If you both confess, you get 5 years each
  - If you both deny, you get 2 years each

Prisoners' Dilemma – equilibrium

		Clyde: Confess	Clyde: Deny
Bonnie	Confess	-5, -5	-1, -10
Bonnie	Deny	-10, -1	-2, -2

- The NE is *(C,C)* and it is ***socially suboptimal***
- Both players would accept to ***collude*** by moving to (D,D) as long as there is ***commitment*** to do so
  - however, this game assumes ***no commitment***
- Players ***prefer*** to receive -5 rather than ***expose*** themselves to the danger of receiving -10

Repeated games

- In repeated games, ***two important things*** may happen:
  1. Players can develop ***reputations***
  2. Players have a chance for ***retaliation***
- Players may ***jointly try to impose*** an outcome that is better than the NE (but not NE)
  - There is ***still incentive for cheating*** in the static game
  - In the repetition process, though, this incentive may be ***eliminated***
  - The cheater can get away with a ***higher payoff*** for ***that*** round
  - ***BUT***, starting from the ***next round***, the player who was cheated upon will ***retaliate*** by choosing the NE strategy

Tit-for-tat strategy

- Retaliation is often called ***tit-for-tat strategy***
  "I trust you and play the collusive strategy but if you cheat, I will be playing the Nash strategy forever"
- The tit-for-tat strategy is a ***trigger strategy***
  everyone trusts everyone else ***until*** someone pushes the trigger
- Once the trigger is ***pushed***
  - The cheater loses ***from next period*** (social optimum → NE)
  - The fair player loses too but can't ***trust*** the cheater anymore
  - For the fair player the NE is better than to be cheated upon
- Can repetition ***lead to*** the ***collusive outcome*** in a prisoners' dilemma type of game?

Repeated games

Indefinite repetition

- What if the game is ***infinitely*** repeated or **randomly terminated**?
- Tit-for-tat strategy ***makes sense***
  - If a player ***cheats***
  - The other player will be playing the ***Nash strategy***, ***forever***
  - The cheater will get high payoff for a ***single period*** but from the ***next period*** will revert to the lower Nash payoff
- The ***threat*** of retaliation can be ***credible***

  iff the one-time payoff from cheating ***does not exceed*** the NPV of the infinite stream of NE payoffs

Repeated games

Sub-perfection

- If a trigger strategy is ***Nash*** for the repeated game starting at period 1, it will be ***also Nash*** for the game starting at ***any period***

  because those games are ***identical***
- In this case, we say that the trigger is a ***sub-game-perfect NE*** (SPNE)

  that is, a NE that ***survives in every*** sub-game that ***includes*** it

Repeated games

Finite repetition

- What if the game is repeated a ***known finite number*** of times?
- Lets take things ***from the end***
  - In the last period there is ***no possibility of retaliation***, thus, everyone will cheat
  - If everyone cheats in the last period, there is ***no fear of retaliation*** to the second to last period
  - So, there is ***no retaliation*** for any period
- The ***threat*** of retaliation is ***not credible***

  collusion is ***not sustainable***

Dynamic games of complete info

- Consider a game between an entrant and an incumbent:

		Entrant (E)	
		Enter	Pass
Inc. (I)	Accommodate	1, 2	2, 0
	Fight	-1, -3	2, 0

- What happens if the entrant ***plays first***?
- The incumbent can ***threat*** the entrant

  if you '*enter*' I will '*fight*' – I may lose 1 but you will lose 3!
- However, this threat will be ***empty***

  once the entrant has entered the incumbent will want to ***accommodate***

Dynamic games

Credibility

- The entrant ***can understand*** from the beginning that the threat is empty

  so, will decide to ***enter***
- This threat has a ***credibility*** problem

  if incumbent ***played first***, things would be ***different***
- ***Dynamic inconsistency:*** A strategy may be optimal ex ante but sub-optimal ex-post, depending on the evolution of the game

  the ***problem*** here is that the NE concept ***cannot distinguish*** empty threats

Dynamic games

Sequential rationality

- The equilibrium concept in a dynamic game must satisfy ***sequential rationality***

  for each player, at every stage, strategy must be ***optimal*** from that node ***and on***
- This seems ***easier*** if
  1. We represent the game in a ***tree form***
  2. Split it in smaller ***sub-games*** from each node
  3. Then, analyze it using ***backward induction***
- ***Zermelo's theorem:*** In every finite game of perfect info backward induction can ***yield*** a NE in pure strategies – if payoffs are asymmetric the equilibrium will be ***unique***

Sub-game perfection

- A ***sub-game*** is a subset of the game that begins with a node and includes ***all consequent*** nodes and branches
  - every sub-game is a game ***on its own***
- ***SPNE:*** A profile of strategies that is a NE in every sub-game of this game that contains this profile
- Intuition
  - The last game is a ***standalone*** game
  - As it results in a unique NE it may be ***replaced*** with its NE
  - This logic is applied then to the ***next-to-last*** game
- Dynamic games are usually represented with a succinct ***game tree***

BMW vs. Benz: product choice problem

- BMW and Mercedes each wish to ***introduce a new type*** of vehicle in the market
  - Either a ***Compact Utility Vehicle*** (CUV)
  - Or a ***Compact Cabriolet*** (Cabrio)
- Firms will be better off if they introduce a ***different type*** of vehicle
  - Because the ***demands*** in those markets are small and ***cannot accommodate*** two competing sellers
  - Plus firms need to sell a ***high quantity*** to reach their MES
- Decisions are ***non – cooperative***

Product choice problem

		Mercedes: CUV	Mercedes: Cabrio
BMW	CUV	-6, -6	12, 10
BMW	Cabrio	10, 12	-5, -5

- If BMW ***hears*** that Mercedes is introducing a CUV, its best action is to produce a Cabrio
- Bottom left corner is ***Nash equilibrium***
- What is ***other*** Nash Equilibrium?

Sequential moves

		Mercedes: CUV	Mercedes: Cabrio
BMW	CUV	-6, -6	12, 10
BMW	Cabrio	10, 12	-5, -5

- If both firms announce their decisions ***independently*** and ***simultaneously***, they may both lose money
- What if Mercedes ***sped up*** production and introduced a new model ***first***?
  - Now there is a ***sequential game***
  - BMW will have to ***produce the opposite*** of what Mercedes produced

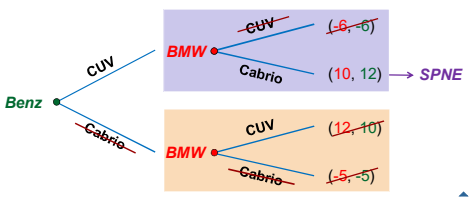
Extensive form

		Mercedes: CUV	Mercedes: Cabrio
BMW	CUV	-6, -6	12, 10
BMW	Cabrio	10, 12	-5, -5

- The above bi-matrix does ***not depict the game with clarity*** anymore
- We have to represent possible moves in the ***extensive form*** of a decision tree
  - this allows to ***work backward*** from the best outcome for BMW

Decision tree

		Mercedes: CUV	Mercedes: Cabrio
BMW	CUV	-6, -6	12, 10
BMW	Cabrio	10, 12	-5, -5



First-mover advantage

- In some games the first-mover (leader) has ***advantage***
  - in the ***previous example*** the company who would chose product first gained 12, while the follower only 10
- The follower can ***threaten*** the leader
  - but an ***empty threat*** will ***not reverse*** the advantage
- Lets now examine a strategy of ***preemptive restriction***

Energon vs. Orange

		Orange (1st)	
		Thin frame	Thick frame
Energon	Thin battery	3, 6	3, 0
	Thick battery	1, 1	8, 3

- Orange Inc. produces ***cellphones***
- Energon produces ***batteries*** for cellphones
- ***Sequential game*** with Orange as the ***first-mover***
- Orange does best by producing ***thin cellphones***
  - Orange ***knows*** that Energon will ***then*** produce thin batteries
- Energon prefers to make ***thick batteries***

Threat

		Orange (1st)	
		Thin frame	Thick frame
Energon	Thin battery	3, 6	3, 0
	Thick battery	1, 1	8, 3

- Can Energon ***induce*** Orange to produce ***thick frames***?
  - ***recall*** that Energon ***moves after*** Orange
- Suppose Energon ***threatens*** to produce thick batteries ***regardless*** what Orange does
  - not credible! – once Orange ***has produced thin*** frames, Energon ***will lose*** if carries out its threat
- Can Energon ***make the threat credible***?

A credible threat

		Orange (1st)	
		Thin frame	Thick frame
Energon	Thin battery	0, 6	0, 0
	Thick battery	1, 1	8, 3

- *Energon* ***burns down (!)*** the production line of small batteries
- *Energon* now can ***credibly threaten*** that it will produce "big batteries"
- Of course, *Energon* wants to make sure that *Orange* executives ***hear*** about the fire ;)

Irrationality

- If a player gets the ***reputation of being "irrational"*** threats might be in fact credible
  - irrational individuals do ***not*** always make ***profit maximizing*** decisions
- In some occasions reputation of irrationality can lead to a ***significant advantage***
  - Opponents ***cannot estimate*** you with logic
  - Your ***threats*** will be taken more ***seriously***

Thank you!

Kosmas Marinakis
www.kmarinakis.org
kmarinakis@hse.ru

WARNING

This printout is provided as a courtesy, so that lecture time can be dedicated to note taking. These slides are **not standalone material** and should be used strictly as **reference**, side by side with notes taken in the lecture. Studying solely from the slides **is not recommended** and might in some cases **mislead** those who have not attended the relevant lecture. **Less than 5% of tasks in tests and exams can be answered from the slides**.