Industrial Economics
ICEF, HSE & UoL
Kosmas Marinakis, Ph.D.
www.kmarinakis.org/ie
https://t.me/icef_industrial

Handout 1

Indexing of variables and other notations

Often in economics we have symmetrical situations for which we use the i, j notation to index our variables without repeating equations or calculations. Consider, for example, two demand curves for firms 1 and 2:

$$q_1 = a - dp_1 + p_2$$
$$q_2 = a - dp_2 + p_1.$$

Both demands can be expressed with a single equation:

$$q_i = a - dp_i + p_j, \quad for \;\; i, j = 1,2 \;\; and \; i \neq j.$$

One may think that the i, j notation does not really save much time or space if one considers just two equations. However, if it is needed to work on those equations, such as calculate profit functions, maximize them and solve for the FOCs, indexing allows us to avoid repeating the calculations for both 1 and 2 but do them once, only for i. This is because i can be 1 and 2, and then, j will be what i is not. I like to think about i as the "random side we consider" and j as the "other side".

The reason we like to use this indexing is because it allows us to omit duplicated calculations without loss of generality or without proving that 1 and 2 are indeed symmetric (which in some cases may not be trivial). Also, the i, j indexing allows entering the equations directly in for-loops when coding. This does not just make the code shorter but also allows the computer to process it much faster (sometimes un-indexed code can run for hours, while indexed only for seconds!).

UoL often uses this notation in exams and all your texts use it often too. Thus, it will be a good idea to familiarize with it. It looks scary and too abstract but in reality is really simple.

Speaking of mathematical notation, here is a brief reminder for other usual notations:

- $\forall$ means 'for every' (for example, $ab > c \quad \forall\, b \in R$)
- : means 'such that' (for example, consider $x^* : f(x^*) = 0$)
- $[a, b]$ is the continuous set that includes all numbers from a to b and $a \leq b$. The set is empty when $a = b$. Sets can include the extremes or not [], (), [), (].
- $\{a, b\}$ is the discrete set that includes only a and b. It is not necessary that $a \leq b$, a, b may not even be numbers (for instance, they may be names such as 'firm a' and 'firm b').
- $\{a, \ldots, b\}$ is the set that includes all integers between a and $b \geq a$.
- Comma within equalities means 'and' (for example, $for \;\; i, j = 1,2$ means 'for i and j taking values 1 and 2' – notice that we use 'for' instead $\forall$ because we do not mean $\forall$)
- $\sim$ means 'follows distribution' (used with random variables). Do not confuse it with $\approx$ which means almost equal.
- | means 'given that' (for example, $p(x|y = 1)$ is $p(x)$ given that $y = 1$).
- A 'parameter' is a number or a variable that is not a control variable (the player cannot optimize it). It is usually something exogenously set.
- $\equiv$ means 'identical' or 'is' and signifies an identity. An identity is an equality that holds always. We use it for definitions. It is different than $=$ in that 'let $x = b - c$' means 'assume that x is equal to $b - c'$, something that may later be contradicted. On the contrary, $x \equiv b - c$ means that $'x$ is defined as the difference $b - c$' it 'is' this difference and it will always be so.

- An ‘identity’ is an equality that is always true – an ‘equation’ means that two different things are at balance – a ‘condition’ is an equality (or other expression) that must hold for something else to happen (conditions do not have to hold always).
- ‘A constraint is binding’ means that it affects the solution of the problem (it actively constrains the outcome). If, for instance, a firm maximizes x under the constraint that $x < 3$ and the solution without taking the constraint into account is $x = 1$, then the constraint is non-binding. If the solution without the constraint was $x = 5$, then the constraint is binding and prevents the solution from being 5.
- $\prec, \succ, \preccurlyeq, \succcurlyeq$ are used to express order of preference. $<, >, \leq, \geq$ are used to express order of size.
- A * superscript does not necessarily signify equilibrium, just a special value for a variable.
- $< x$ always means ‘smaller than x’ unless $x = 3$. If $x = 3$, then it does not necessarily mean ‘smaller than 3’.
- :) does not mean ‘such that the upper extreme is excluded’.