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Lecture 3

Contracts

Industrial Economics

TRIGGER WARNING

The following presentation contains images and descriptions of *graphic violence* that some audiences may find disturbing

Discretion is advised

Economic rationality

- On February 17th 2001 notorious Greek robber ***Kostantinos Passaris*** is transferred from prison to hospital for scheduled exams
- During the wait someone passes to the criminal a loaded ***9mm pistol***
- Within a moment Passaris ***kills*** two of the guards and ***seriously injures*** the third one
- While fleeing the scene, he comes ***face to face*** with the driver of the police van who was waiting at the yard

  "I will not kill you because the bullet is worth more than your […] life"

Von Neumann – Morgenstern rationality

Consider 3 lotteries L, M, N

1. ***Completeness***

   L, M, N can be ranked using the relationships $\prec, \preccurlyeq, =$
2. ***Transitivity***

   $L \preccurlyeq M$ and $M \preccurlyeq N \;\Rightarrow\; L \preccurlyeq N$ (preferences are consistent)
3. ***Continuity***

   $L \preccurlyeq M \preccurlyeq N \;\Rightarrow\; \exists\, p \in [0,1] \;:\; pL + (1-p)N = M$

   (a mix of the worst and the best can yield the middle option)
4. ***Independence***

   $L \prec M \;\Rightarrow\; pL + (1-p)N \prec pM + (1-p)N,\ \forall N \,\&\, \forall\, p\ (0,1]$

   (same mixtures of irrelevant lotteries cancel out)

Rational preferences

- Preferences are ***rational*** when they obey the VNM axioms
- The Von Neumann–Morgenstern ***theorem***

  for any VNM-rational agent, there exists function u mapping any lottery to a real number such that

  $$\forall\, M \prec N \;\Rightarrow\; u(M) < u(N)$$

The profit maximization hypothesis

- A ***common assumption*** in economic theory is that firms maximize (expected) profits
  - This is probably what the ***owners*** would like to do
  - But, most firms are ***not run*** by the owners
- Managers are likely to have ***other objectives*** than profit maximization
- How ***tangible*** is profit maximization?

Deviation of interests

- There exists an ***asymmetry*** problem
- Managers:
  - Have more ***information*** for the operations
  - May incur different ***costs*** than owners
  - May have different ***risk attitude*** or ***time preference*** than owners
- Principal and agent have ***misaligned interests***

  but the one with the ***less authority*** has ***more information***
- This deviation of interests is an ***informational problem***

  managers have the opportunity to take ***hidden action*** avoiding, fooling or corrupting ***monitoring***

Adverse selection

- Asymmetry in information ***prior to the deal***
  - The better informed party will ***selectively participate*** in advantageous trades and ***withdraw*** from disadvantageous
  - The less informed party will ***incorporate*** the lack of information in its expectations for the outcome of the trade
- S and B are interested in trading a good of ***value*** V
  - S knows that $V = 1$
  - B knows that $V \sim Uniform[0,1]$
- B is ***not willing to pay*** more than 0.5 for the good

  thus, S ***will withdraw*** from an otherwise ***efficient*** deal
- ***Remedy*** for adverse selection is ***screening*** or ***menus***

Moral hazard

- Asymmetry in information ***after the deal***
- Actions of one party may ***change*** to the detriment of another after the deal has taken place
- Moral hazard has ***two aspects***:
  1. One party may take ***more risks*** because the other bears the ***cost*** of those risks
  2. One party may take ***hidden action***
- ***Examples:*** car insurance, labor contracts

- Adverse selection deals with the ***agent's type*** while moral hazard deals with the ***agent's actions***

A discrete contract

- An risk neutral owner (principal - she) hires a risk averse manager (agent - he) for a ***salary*** w
- The agent chooses his ***effort:*** $e \in \{0, e^*\}$
- Agent's ***utility*** is:

  $$u(w-e), \quad \partial u/\partial w > 0, \quad \partial^2 u/\partial w^2 < 0$$

  that is, effort causes e units of ***monetary disutility***
- Agent's ***reservation*** wage is: $w_o : u(w_o) = u_o$
- Principal is risk neutral and receives ***gross profit***

  $\pi \in \{\pi_L, \pi_H\}$, where $\pi_L < \pi_H$
- The realization of π depends ***stochastically*** on the agent's effort choice, e

Stochasticity

- To simplify the notation we ***define***

$p_{H,e^*} \equiv p(\pi = \pi_H \vert e = e^*)$	$p_{L,e^*} \equiv p(\pi = \pi_L \vert e = e^*)$
$p_{H,0} \equiv p(\pi = \pi_H \vert e = 0)$	$p_{L,0} \equiv p(\pi = \pi_L \vert e = 0)$

- For $\boldsymbol{e = e^*}$, $\quad p_{H,e^*} + p_{L,e^*} = 1$
- For $\boldsymbol{e = 0}$, $\quad p_{H,0} + p_{L,0} = 1$
- It must be $\quad p_{H,e^*} > p_{H,0}$

  increasing effort ***must increase*** the probability of π_H
- Analogously $\quad p_{L,0} > p_{L,e^*}$

  decreasing effort ***must increase*** the probability of π_L
- This property is known as ***stochastic dominance***

Stochastic dominance

- Stochastic dominance does not imply that the ***most probable*** result under high effort would be success (π_H)

  that is, $p_{H,e^*} > p_{L,e^*}$ may ***not*** be necessarily true
- Imagine a ***very difficult*** project in which
  - $e = 0$ gives the project 1% chance to succeed
  - $e = e^*$ gives the project 2% chance to succeed
- The setting ***satisfies*** stochastic dominance: $\overset{2\%}{p_{H,e^*}} > \overset{1\%}{p_{H,0}}$
- But does ***not satisfy*** that $\overset{2\%}{p_{H,e^*}} > \overset{98\%}{p_{L,e^*}}$

The contract

- The principal wants to come up with a ***compensation scheme*** to ***incentivize*** the agent to ***exert*** the amount of effort ***she wants***
- The principal can make w ***contingent*** on some condition
- Such as,

$$w(e) = \begin{cases} w_1, & if\ e = 0 \\ w_2, & if\ e = e^* \end{cases}$$

the condition may involve ***any variable*** that the principal can ***observe*** and ***verify (contractible)***

- The principal will ***offer take-it-or-leave-it*** the scheme to the agent ***before*** he chooses e

Contractible effort

- The contract will be contingent on ***effort***
- Assuming that the principal wants to ***implement*** e^*, the scheme would simply be:

$$w(e) = \begin{cases} 0, & if\ e = 0 \\ w_o + e^*, & if\ e = e^* \end{cases}$$

- Assuming that the principal wants to ***implement zero effort***, the scheme would be:

$$w(e) = \begin{cases} w_o, & if\ e = 0 \\ 0, & if\ e = e^* \end{cases}$$

if she offers flat w_o the agent will again select $e = 0$

Unobservable effort

- The contract now ***cannot*** be contingent on effort
- It can be contingent on ***profit***

π ***stochastically*** depends on agent's ***costly*** choice, e

- The principal faces an ***informational asymmetry*** (***?***)

- moral hazard!

- Contract will ***look like***:

$$w(\pi) = \begin{cases} w_L, & if\ \pi = \pi_L \\ w_H, & if\ \pi = \pi_H \end{cases}$$

- The principal is ***risk neutral*** and ***maximizes*** $E(\pi - w)$

must set $w(\pi)$, so that the agent will ***accept*** the offer and ***exert*** the effort required by the principal

Incentive conditions

- In general, ***any*** incentive problem is governed by two basic ***conditions*** which should hold at equilibrium

the ***effort level*** that the principal wants to ***implement***

1. Individual rationality (IR)

$$Eu(w, e^*) \geq u_o$$

that is, at e^* the expected utility of the agent should be at least as good as the ***outside option***

2. Incentive compatibility (IC)

$$Eu(w, e^*) \geq Eu(w, e') \quad \forall\ e' \neq e^*$$

that is, at e^* the expected utility of the agent should be at least as good as the expected utility ***of any other effort choice***

IC and IR for $e = e^*$

- The principal must calculate the agent's ***expected utility*** for each possible effort level:
  - $EU_{e=0} = p_{L,0} \cdot u(w_L) + (1 - p_{L,0}) \cdot u(w_H)$
  - $EU_{e=e^*} = p_{H,e^*} \cdot u(w_H - e^*) + (1 - p_{H,e^*}) \cdot u(w_L - e^*)$
- The agent will ***sign the contract*** iff

$$EU_{e=e^*} \geq u_o \qquad \text{(IR)}$$

- The agent will ***exert*** e^* iff

$$EU_{e=e^*} \geq EU_{e=0} \qquad \text{(IC)}$$

- The agent will sign the contract even if the IR and IC hold with ***equality*** – no reason for ***rents***

The optimal scheme for $e = e^*$

- The principal's ***expected payoff*** is

$$E\Pi_{e=e^*} = p_{H,e^*} \cdot (\pi_H - w_H) + (1 - p_{H,e^*}) \cdot (\pi_L - w_L)$$

- The principal's ***problem*** becomes

$$\max_{w_L, w_H} E\Pi_{e=e^*}$$

s.t. $EU_{e=e^*} = u_o$ and $EU_{e=e^*} = EU_{e=0}$

- The combination (w_H^*, w_L^*) which solves the principal's problem will ***satisfy***:

$$w_H^* > w_L^*$$

the agent is ***rewarded*** if profit turns out high, no matter the actual effort

Comparison of Ew when $e = e^*$

- Expected wage for the agent under unobservable effort:

$$Ew_{un} = p_{H,e^*} \cdot w_H^* + (1 - p_{H,e^*}) \cdot w_L^*$$

- Under observable effort:

$$Ew_{ob} = w_o + e^*$$

- Comparison

$$Ew_{un} > Ew_{ob}$$

  - Agent is ***risk averse*** – needs to receive ***risk premium*** for the case that effort is high but profit low
  - The principal's ***net profit*** will be ***lower*** under unobservable effort

Unobservable effort and $e = 0$

- The principal's problem:

$$\max_{w_L, w_H} E\Pi_{e=0}$$

s.t. $EU_{e=0} = u_o$ and $EU_{e=0} = EU_{e=e^*}$

- The IC is ***not binding*** in this case

if the agent is offered wage for $e = 0$ but deviates to $e = e^*$ with the same wage, the principal does ***not*** mind because

$$E\Pi_{e=e^*} > E\Pi_{e=0}$$

- The IR is ***still binding***

the principal will offer a flat w_o to ***just ensure participation***

- ***Expected payoff*** for the principal is

$$E\Pi_{e=0} = p_{L,0} \cdot (\pi_L - w_o) + (1 - p_{L,0}) \cdot (\pi_H - w_o)$$

$e = e^*$ or e = 0?

- The ***choice*** to implement $e = 0$ or $e = e^*$ will depend on the ***comparison*** of $E\Pi_{e=0}(w_o)$ and $E\Pi_{e=e^*}(w_L, w_H)$

- Notice that the entire derivation of the contract is based on ***backward induction***

- The ***principal*** derives the ***reaction*** of the agent to the contractual parameters

then she sets the appropriate scheme of w to ***manipulate*** him

ευχαριστώ!

(thank you!)

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