Kosmas Marinakis, Ph.D.

Lecture 13

Game Theory – part II

micro2 first module m2

Rock – Scissors – Paper

		Player B		
		Rock	Scissors	Paper
Player A	Rock	0 , 0	1 , -1	-1 , 1
	Scissors	-1 , 1	0 , 0	1 , -1
	Paper	1 , -1	-1 , 1	0 , 0

- There is ***no NE*** in pure strategies

  no combination of strategies that some player does not want to ***deviate unilaterally*** from
- Then, what is the ***best strategy (equilibrium)*** for this game?

Mixed strategies

- Sometimes the best strategy is ***not a pure strategy***
- Players have to do ***randomization***
- That is, to play a ***mixed strategy***

  assign a ***probability*** to every available strategy
- ***Example:*** 30% *up*; 20% *middle*; 50% *down*;
- The ***actual strategy*** that will be played is chosen from the mix ***randomly*** based on the assigned probabilities
- A combination of mixed strategies is a ***NE equilibrium*** if no player has an incentive to change the mix of probabilities unilaterally

Mixed strategies NE in R-S-P

- In the R-S-P game the ***NE is in mixed*** strategies

  randomize (or mix) all strategies with probability 1/3
- At the NE, both players will be doing ***the best*** they can ***given*** what their opponent is doing
- If you play any other strategy or mixture, your rival may realize it and ***play accordingly*** to ***take advantage*** of you
- What happens if you play ½ Rock and ½ Scissors?

  your opponent will ***keep playing Rock*** and you will never win!

Some useful facts

1. John Nash proved that ***every static game*** has ***at least one*** NE either in pure or in mixed strategies
2. A game might have ***both pure and mixed*** strategy NE
3. The calculation of the optimal probabilities for the randomization in a mixed strategy NE involves ***optimal response functions***
4. Mixed strategies are usual in games like ***poker*** – ***real firms*** might not find it reasonable to play mixed strategies

Methodology

		P2		
		C	D	
P1	A	0 , 1	1 , 0	p
	B	1 , 0	0 , 2	$1-p$
		q	$1-q$	

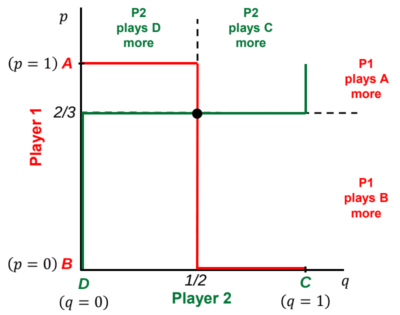
- For P1:

$$\left.\begin{array}{l} E\Pi_A = 0\cdot q + 1\cdot(1-q) = 1-q \\ E\Pi_B = 1\cdot q + 0\cdot(1-q) = q \end{array}\right] \Rightarrow 1-q = q \Rightarrow \boxed{q=\frac{1}{2}}$$

- For P2:

$$\left.\begin{array}{l} E\Pi_C = 1\cdot p + 0\cdot(1-p) = p \\ E\Pi_D = 0\cdot p + 2\cdot(1-p) = 2-2p \end{array}\right] \Rightarrow p = 2-2p \Rightarrow \boxed{p=\frac{2}{3}}$$

Mixed strategies – graph



Repeated games

- Consider a ***static game*** which is ***repeated*** again and again
- ***Oligopolistic firms*** often play a repeated game
  - they compete for ***more than one periods***
- When games are repeated, ***two important things*** may happen:
  1. Players have a chance for ***retaliation***
  2. Players can develop ***reputations***

Sustainability of non-Nash outcomes

- The firms can decide to ***collude***
  - implement an outcome ***better than the NE*** (but ***not a NE***)
- Collusion is ***not stable***
  - players have an ***incentive to deviate*** (cheat)
- If a player decides to cheat, he can get away with a higher profit ***for that period***
- BUT, starting ***from the next period***, the player who was cheated upon will retaliate by choosing his NE strategy

Pricing problem

		PlayStation	
		Low price	High price
Xbox	Low price	10, 10	100, -50
	High price	-50, 100	50, 50

- NE implies that they ***both set low prices***
- Collusion is ***better*** than NE but there is ***incentive*** for ***cheating***
  - it is ***even better*** for a firm to set a low price while the other firm sets a high price
- Collusion ***may be sustained*** if the game is repeated
  - firms might adopt a ***tit-for-tat strategy***

Tit-for-Tat strategy

- Retaliation is often called ***tit-for-tat*** strategy
  - *I trust you and play the collusive strategy but **if you cheat**, I will be playing my Nash strategy forever*
- The tit-for-tat strategy is a ***trigger strategy***
  - everyone trusts everyone else ***until*** someone… pushes the trigger
- Once the ***trigger is pushed***
  - The cheater loses from ***next period*** (collusion → NE)
  - The fair player loses too but ***does not trust*** the cheater anymore
  - For the fair player the NE is better than to be ***cheated upon***

Indefinite repetition

- What if the game is infinitely repeated?
- Competitors repeatedly set price ***every period***, forever
- ***Tit-for-tat*** strategy makes sense
- If a player ***cheats***:
  - The other player will be playing the ***Nash strategy***, forever
  - The cheater will get high profits ***for that period*** but from the ***next one*** will be getting much less
- The threat of retaliation is ***credible*** and may ***prevent*** players from cheating
  - if the ***cheating payoff*** is ***exceeded*** by the ***NPV of future*** collusion payoffs

Finite repetition

* What if the game is repeated a ***known finite number*** of times?
* Lets take things ***from the end***
  * In the last period there is ***no possibility of retaliation***, thus, everyone will cheat
  * Yes, but if everyone cheats in the last period, there is ***no fear of retaliation*** to the second to last period
  * So, there is ***no possibility of retaliation*** for any period
* The ***threat*** of retaliation is ***not credible***

  collusion is ***not sustainable***

Cooperation in repeated games

* Cooperation is ***at best difficult***
  * ***Conditions may change*** in the long-run
  * Need a ***small number*** of firms
  * Need ***stable demand*** and ***cost*** conditions
* Sometimes, a firm might have a ***legitimate reason*** to lower price and avoid to do it

  fear that such action may be ***misunderstood*** and push ***accidentally*** the trigger

Sequential games

* In sequential games players ***move in turns***, responding to each other's actions and reactions
  * Ex: ***Stackelberg*** model
  * Responding to a competitor's ***ad campaign***
  * ***Entry*** decisions

BMW vs. Benz – revisited

BMW \ Mercedes	CUV	Cabrio
CUV	-6, -6	12, 10
Cabrio	10, 12	-5, -5

* If both firms announce their decisions ***independently*** and ***simultaneously***, they may ***both lose money***
* What if Mercedes ***sped up*** production and introduced a new model ***first***?
  * Now there is a ***sequential game***
  * BMW will have to ***produce the opposite*** of what Mercedes produced

Extensive form

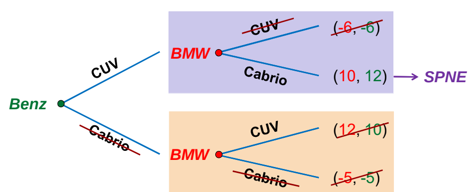
BMW \ Mercedes	CUV	Cabrio
CUV	-6, -6	12, 10
Cabrio	10, 12	-5, -5

* The above bi-matrix ***does not depict*** the game with clarity anymore
* We have to represent possible moves in the ***extensive form*** of a ***decision tree***

  we can ***work backward*** from the best outcome for BMW

Decision tree

BMW \ Mercedes	CUV	Cabrio
CUV	-6, -6	12, 10
Cabrio	10, 12	-5, -5



Sub-game Perfect NE

- In the previous product-choice game we ***split*** the game into sub-games
- Then we found the NE in ***every*** sub-game
- ***Sub-game Perfect NE*** (SPNE): A combination of strategies which is a NE ***in every subsequent sub-game*** that includes this combination
- We will use the SPNE as the ***basic*** equilibrium ***notion*** in dynamic games

The first-mover advantage

- In the previous product-choice game, there is a clear ***advantage to moving first***
- In ***quantity competing oligopoly*** there is the ***same*** advantage
  - The firm which goes first can choose a ***large level of output***, thereby ***forcing the second firm*** to choose a small level
  - ***Compare*** Cournot vs. Stackelberg

How to make the first move

- Demonstrate ***commitment***
- If BMW wants to move first it ***has to commit***
  - An announcement that it will produce a CUV is ***not enough***
  - It can invest in ***expensive advertising*** campaign
  - Place a large ***order of CUV tires*** and… send invoice to Mercedes
- Commitment must be ***serious enough*** to ***induce*** Mercedes to make the decision BMW wants it to make

ευχαριστώ!

(thank you!)

Kosmas Marinakis
www.kmarinakis.org
kmarinakis@hse.ru
kosmas_marinakis
Kosmas Marinakis
t.me/kosmas_teaching
@Kos_Marinakis