## Practice Set 6

## Game Theory

This problem set contains material for the relevant lab. Lab teachers are expected to provide sufficient guidance for the entire problem set. It is in the teacher's discretion to select the most representative tasks to solve instructionally in every lab. For the rest of the tasks, methodology, hints and final answers will be provided. Students are expected to work on practice problems, however, they are not required to submit written solutions. It is a non-negotiable policy in this course to not provide handouts with the solutions of practice problem sets.

1. Find the NE in the following simultaneous game between player 1 and player 2.

| $\mathrm{P} 1 \backslash \mathrm{P} 2$ | L | R |
| :---: | :---: | :---: |
| T | 2,1 | 0,2 |
| B | 1,2 | 3,0 |

2. Find the NE in the following simultaneous game between player 1 and player 2.

| $\mathrm{P} 1 \backslash \mathrm{P} 2$ | L | R |
| :---: | :---: | :---: |
| T | $2,-3$ | 1,2 |
| B | 1,1 | $4,-1$ |

3. Find the NE in the following simultaneous game between player 1 and player 2.

| $\mathrm{P} 1 \backslash \mathrm{P} 2$ | L | R |
| :---: | :---: | :---: |
| T | 1,0 | 0,1 |
| B | 0,1 | $\mathrm{a}, 0$ |

4. Consider the following game by-matrix for a dynamic game.

| $\mathrm{P} 1 \backslash \mathrm{P} 2$ | L | R |
| :---: | :---: | :---: |
| T | 10,1 | $-3,3$ |
| B | 1,2 | $7,-1$ |

(a) Find the SPNE if P1 moves first.
(b) Find the SPNE if P2 moves first.
(c) If we could auction the priority of moving, who would bid on it and how much?
5. Consider the following game:

(a) Find the equilibrium
(b) Can you suggest a Pareto improvement relative to the equilibrium outcome in (a)? Why may it be hard to achieve it?
6. Consider the following game:

| $\mathrm{P} 1 \backslash \mathrm{P} 2$ | I | N |
| :---: | :---: | :---: |
| I | 1,1 | $0, c$ |
| N | $c, 0$ | $c, c$ |

(a) Solve the game assuming $0<c<1$.
(b) Knowing that strategy "I" corresponds to "invest" action and "N" corresponds to "not invest", what is the economic interpretation of this game?
(c) Which strategy is less risky and why?
7. Find the NE in mixed strategies in the following simultaneous game between player 1 and player 2. Explain why this is a NE.

| $\mathrm{P} 1 \backslash \mathrm{P} 2$ | $L$ | $R$ |
| :---: | :---: | :---: |
| $T$ | 0,1 | 1,0 |
| $B$ | 1,0 | 0,2 |

8. Firm 1 and firm 2 have one job opening each and they offer wages $w_{1}$ and $w_{2}$ such that $0.5 w_{1}<w_{2}<2 w_{1}$. There are two risk-neutral workers, each of whom can apply to only one firm. Workers will simultaneously decide to which firm they will apply. If only one worker applies to a given firm, that worker gets the job; if both workers apply to the same firm, one worker is hired at random with probability (i.e. with probability of $50 \%$ ) and the other worker remains unemployed and gets zero payoff.
(a) Find the NE in pure strategies in the game that the two workers play.
(b) Is there a NE in mixed strategies?
(c) How does the spread between $w_{1}$ and $w_{2}$ affect the NE in this game?
9. Five pirates found 100 gold coins and need to decide how to distribute them. The pirates are ranked by strict seniority from the most senior one to the least senior one. The most senior pirate is the first to make an offer. If at least one half of pirates (including the one who made the proposal) accept this offer, coins are distributed accordingly. Otherwise the most senior pirate is executed and a new offer is made by the next most senior pirate. The same rules apply until a decision is taken. Predict the outcome of this game.
10. Find the IDE in the following simultaneous game between player 1 and player 2.

| P1 \P2 | a | b | c |
| :---: | :---: | :---: | :---: |
| A | 18,18 | 15,20 | 9,18 |
| B | 20,15 | 16,16 | 8,12 |
| C | 18,9 | 12,8 | 0,0 |

