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# Lecture 10

Oligopoly – part II

micro2
first module

m2



## Actions & reactions

- In all <u>other market structures</u> every firm is simply ***doing its best*** no matter what others do
- This is ***not the case*** in <u>oligopoly</u>

  because everyone's ***outcome*** is affected by everyone's ***actions***
- Every firm is doing their best ***given what it believes*** that its competitors will do
- We call this kind of behavior ***strategic behavior***
- Actions and reactions may be ***dynamic***

  may ***evolve*** over time





## Equilibrium concept

- In general, ***equilibrium*** is a state, from which there is <u>no tendency for deviation</u>
- So far, we have used a ***strong notion*** of equilibrium

  *"a state from which no one has a tendency to deviate <u>in any way or fashion</u>"*
- We ***cannot use*** this concept any more
  - It will ***not work*** in most of the cases
  - When there is interaction if I ***deviate*** from my strategy, I ***affect*** your outcome, too





## Nash Equilibrium

- Instead, we will use a ***more general*** but ***also weaker*** notion of equilibrium, the ***Nash Equilibrium*** (NE)
- Each firm follows a ***strategy***
  - That is, ***selects one action*** from a set of possible actions
  - When each firm selects its strategy, we have a ***combination of strategies***
- A combination of strategies is NE, when <u>no firm has an incentive to *unilaterally* deviate</u> from this combination

  no firm has ***something to gain*** by changing ***only their own*** strategy





## Facts at the NE

- Each firm selects the strategy that ***maximizes*** its profit considering its ***belief*** on what the other firms will do
- Beliefs for what the competitors are doing are ***correct***
- Each firm is doing the ***best*** it can ***given*** what other firms are doing
- If one firm ***alone*** changes its strategy, it will do ***worse***



## The Cournot duopoly (1838)

- Two firms produce a ***homogeneous or heterogeneous*** good
- They ***simultaneously*** decide ***how much*** to produce
  1. That is, their ***choice variable*** is $q$
  2. They decide at the ***same time***
  3. Decisions are ***irrevocable***
- Each firm will ***adjust its quantity*** based on what ***it thinks*** the other will produce
  - Each firm will treat the quantity of the rival firm as ***<u>a</u> constant***
  - That is, ***not*** as a choice variable

## Cournot duopoly model

* The two firms face ***linear market demand*** curve

$$p = a - bq_1 - bq_2$$

* Both firms have ***constant marginal cost***, $c$
* ***Profits*** for the firms are

$$\Pi_1 = (a - bq_1 - bq_2)q_1 - cq_1 = (a - c - bq_2)q_1 - bq_1^2$$

$$\Pi_2 = (a - bq_1 - bq_2)q_2 - cq_2 = (a - c - bq_1)q_2 - bq_2^2$$





## Choice of quantity for firm 1

* Firm 1 will attempt to ***maximize*** $\Pi_1$ with respect to $q_1$ treating $q_2$ as a ***constant***

$$\frac{\partial \Pi_1}{\partial q_1} = 0 \Rightarrow a - c - bq_2 - 2bq_1 = 0 \Rightarrow q_1^* = \frac{a - c - bq_2}{2b}$$

* ***Ooops !!!***
  the optimal $q_1^*$ ***depends*** on $q_2$!
* This is the ***interaction***
* ***Solve*** the FOC for firm 1 with respect to $q_2$

$$q_1^* = \frac{a - c - bq_2}{2b} \Leftrightarrow q_2 = \frac{a - c}{b} - 2q_1^* \quad (R1)$$





## Choice of quantity for firm 2

* ***Firm 2*** runs into the ***same situation*** of interaction

$$\frac{\partial \Pi_2}{\partial q_2} = 0 \Rightarrow a - c - bq_1 - 2bq_2 = 0$$

* Again, the optimal $q_2^*$ ***depends*** on $q_1$
* ***Solve*** the FOC for firm 2 with respect to $q_2$, too

$$q_2 = \frac{a - c}{2b} - \frac{1}{2}q_1 \quad (R2)$$





## Optimal reactions

* (R1) and (R2) are known as ***reaction functions*** or ***optimal response functions***
* (R1):
  *"give me your $q_2$ to tell you which $q_1$ maximizes my $\Pi_1$"*
* (R2):
  *"give me your $q_1$ to tell you which $q_2$ maximizes my $\Pi_2$"*
* (R1) is a function that shows the ***optimal reaction*** of firm 1 to actions by firm 2
* (R2) is a function that shows the ***optimal reaction*** of firm 2 to actions by firm 1





## Cournot – Nash Equilibrium

* Each firm's reaction function tells it how much ***is optimal*** to produce ***for every*** quantity its competitor may choose
* Each firm will ***decide how much*** to produce by
  1. ***Assuming*** how much its rival will produce ($q_2$)
  2. ***Plugging*** this to its optimal response ($q_1^*(q_2)$)
* If ***both firms*** follow this process
  - The equilibrium is at the ***intersection*** of the reaction curves
  - We can ***solve the system*** of (R1) and (R2) to find $q_1^*$ and $q_2^*$
  - The ***NE*** is the combination: $(q_1^*(q_2^*), q_2^*(q_1^*))$
* At the ***NE*** each firm ***correctly assumes*** how much its competitor will produce





## Equilibrium in Cournot model

* ***Solving the system*** of the two reaction curves

$$q_1^* = q_2^* = \frac{a - c}{3b}$$

* From the ***demand curve***

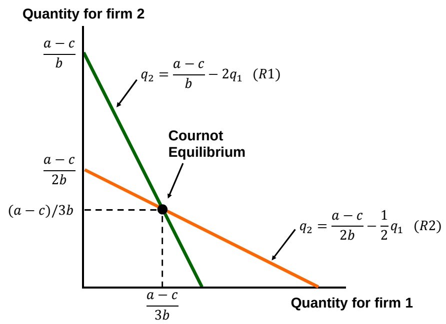
$$p = a - bq_1^* - bq_2^* = a - b\frac{a - c}{3b} - b\frac{a - c}{3b} \Rightarrow p^* = \frac{a + 2c}{3}$$

* ***Profit for each firm*** is

$$\Pi_1^* = \Pi_2^* = \frac{(a - c)^2}{9b}$$

* ***Total profit*** in the industry is $\Pi = 2\frac{(a-c)^2}{9b}$

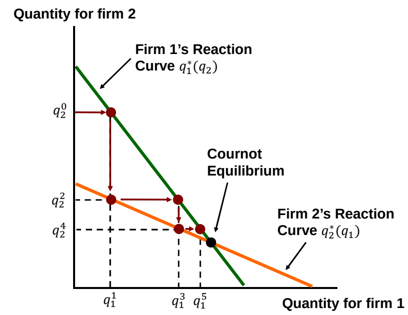
## Reaction curves & equilibrium








## Convergence to Cournot equilibrium








## Static adjustment process

- Cournot equilibrium is ***an instance*** of a Nash equilibrium
- In the way we have defined this notion previously it is obviously ***static***
- The Cournot equilibrium ***says nothing*** about the ***dynamics*** of the adjustment process

  since both firms adjust their output, ***neither output*** would be ***fixed***



## Collusion

- Competition "***eats away***" firms' profits

  both firms ***give up*** market power as they try to gain market share
- Perhaps, it would be profitable for both firms to ***stop being aggressive*** and share the market by ***forming a cartel***
- ***Collusion*** would allow firms to behave as a ***monopoly***
  - Increase the ***joint profit*** and then share it
  - Firms will share the profit ***according*** to relative ***bargaining power***
  - If firms have ***different costs***, the cartel will behave as a ***multi-plant monopoly***





## Collusion model

- ***Demand*** is again $p = a - bq_1 - bq_2 \;\; or \;\; p = a - b \cdot (q_1 + q_2)$
- ***Joint profit***, then, is

$$\Pi_J = [a - b \cdot (q_1+q_2)] \cdot (q_1+q_2) - c \cdot (q_1+q_2)$$

- We treat $(q_1+q_2)$ as a ***single variable*** and ***maximize*** $\Pi_J$

$$\frac{d\Pi_J}{d(q_1+q_2)} = 0 \;\Rightarrow\; (q_1+q_2)^* = \frac{a-c}{2b}$$

- We can ***plot*** this quantity as the ***contract curve***

$$q_2^* = \frac{a-c}{2b} - q_1^*$$

***shows*** all ***combinations*** of output that maximize total profits





## Collusion model equilibrium

- Assuming that the two firms have ***equal bargaining power***, total quantity will be ***shared evenly*** between firms

$$q_1^* = q_2^* = \frac{a-c}{4b}$$

- From the ***demand*** curve, price will be

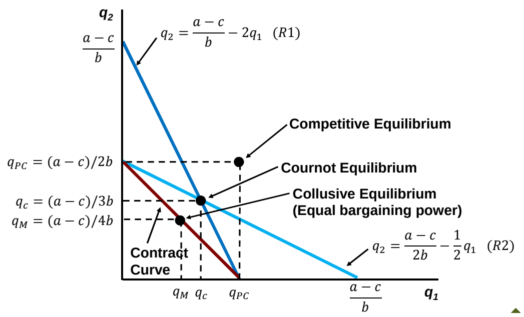
$$p^* = \frac{a+c}{2}, \text{ (same as monopoly)}$$

- ***Joint profit*** will be

$$\Pi_J^* = \frac{(a-c)^2}{4b}, \text{ (same as monopoly)}$$

under equal bargaining power firms will ***share*** $\Pi_J^*$ ***equally***, too

## Cournot vs. collusion vs. PC - graph





## Cournot vs. collusion vs. PC

- Lets ***assume*** that

$$c = 1, \quad b = 1, \quad a = 10$$

- In PC

$$q_1^* = q_2^* = 4.5, \quad p^* = 1, \quad \pi_1^* = \pi_2^* = 0$$

- In Cournot competition

$$q_1^* = q_2^* = 3, \quad p^* = 4, \quad \pi_1^* = \pi_2^* = 9$$

- In collusion (cartel)

$$q_1^* = q_2^* = \frac{a-c}{4b} = 2.25, \quad p^* = \frac{a+c}{2} = 5.5,$$

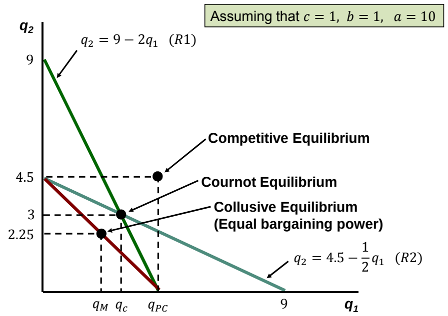
$$\pi_1^* = \pi_2^* = \frac{(a-c)^2}{8b} = 10.125$$



## Cournot vs. collusion vs. PC - example

Assuming that $c = 1,\ b = 1,\ a = 10$





## Stability of collusion

- It is ***obviously better*** for both firms to collude
  - each firm to produce $q_M/2$, $p^* = p_M$ and $\Pi^* = \Pi_M$
- ***Yet***, this outcome is no one's ***best response***
  - therefore, it is not ***a NE***
- Each firm has an ***incentive*** to produce ***more than*** $q_M/2$, if the other firm produces $q_M/2$
  - increase in $q$ will yield ***higher profits*** because $\Delta p < \Delta q$
- ***However,*** this is the case if ***only one*** firm deviates
  - one-way deviation is usually referred to as ***cheating***
- If they ***both cheat***, they revert to ***Cournot***

## Stackelberg leadership (1934)

- Bring in your mind a ***Cournot duopoly*** situation
- Now assume that one of the firms has the ***first-mover-advantage***
  - One firm can ***choose*** its output ***before*** the other firm has a chance to do so
  - This creates a ***leader*** firm and a ***follower*** firm
- When the follower makes his output decision, he ***can see*** how much the leader ***has already produced***
- The leader ***can assess*** the reaction of the follower and thus, can ***take it into account*** in her output decision



## Stackelberg model

- Assume that firm 1 is the ***leader*** and that ***demand*** is

$$p = a - bq_1 - bq_2$$

- The ***reaction*** of the follower is ***identical*** to that in Cournot

$$q_2 = \frac{a-c}{2b} - \frac{1}{2}q_1$$

- The ***leader's profit*** is

$$\Pi_1 = (a - bq_1 - bq_2) \cdot q_1 - c \cdot q_1$$

- The leader ***knows how the follower reacts*** and ***can use*** this info in her profit function

$$\Pi_1 = \frac{a-c}{2}q_1 - \frac{1}{2}bq_1^2$$

## Stackelberg equilibrium

- The leader ***maximizes*** her profit

$$\frac{\partial \Pi_1}{\partial q_1} = 0 \;\Rightarrow\; q_1^* = \frac{a-c}{2b}, \qquad \text{(same } q_1^* \text{ as in PC)}$$

- The follower ***responds*** to $q_1^*$ according to his reaction

$$q_2^* = \frac{a-c}{4b}, \qquad \text{(same } q_2^* \text{ as in Monopoly)}$$

- The ***demand*** yields $p$ for the combination $(q_1^*, q_2^*)$

$$p^* = \frac{a+3c}{4}, \qquad (p^* \text{ between PC and Monopoly})$$

- Always, the leader is ***better off*** and the follower ***worse off***
  - The leader sells ***more*** than in Cournot – the follower sells ***less***
  - The ***price is lower*** than Cournot





## Stackelberg model – conclusions

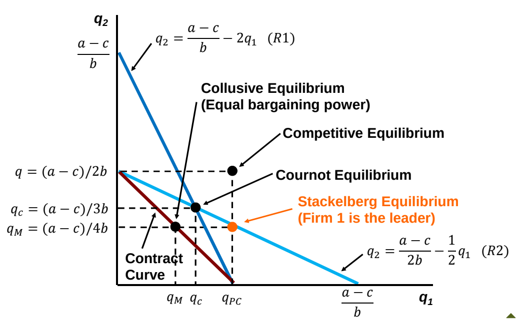
- The ***only advantage*** that the leader has, is that she ***moves first***
  - Leader's ***output*** is twice as large as follower's
  - Leader's ***profit*** is twice as large as follower's
- ***Going first*** allows the leader to produce a larger quantity

  if the follower does not produce less than the leader, profits will be reduced ***for both of them***
- Is the Stackelberg model a ***dynamic*** model?





## *Comparison of models








## *Comparison of models – example

Assuming that $c = 1,\ b = 1,\ a = 10$

| Model | $p$ | $q_1$ | $q_2$ | $q_T$ | $\Pi_1$ | $\Pi_2$ | $\Pi_T$ |
|---|---|---|---|---|---|---|---|
| PC | 1 | 4.5 | 4.5 | 9 | 0 | 0 | 0 |
| Cournot | 4 | 3 | 3 | 6 | 9 | 9 | 18 |
| Stackelberg | 3.25 | 2.25 | 4.5 | 6.75 | 5.06 | 10.125 | 15.19 |
| Collusion | 5.5 | 2.25 | 2.25 | 4.5 | 10.125 | 10.125 | 20.25 |



ευχαριστώ!

*(thank you!)*

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