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Abstract A celebrated result in the theory of tournaments is that relative performance evaluation (tournaments) is a superior compensation method to absolute performance evaluation (piece rate contracts) when the agents are risk-averse, the principal is risk-neutral or less risk-averse than the agents and production is subject to common shocks that are large relative to the idiosyncratic shocks. This is because tournaments get closer to the first best by filtering common uncertainty. This paper shows that, surprisingly, tournaments are superior even when agents are liquidity constrained so that transfers to them cannot fall short of a predetermined level. The rationale is that, by providing insurance against common shocks through a tournament, payments to the agents in unfavorable states increase and payments in favorable states decrease which enables the principal to satisfy tight liquidity constraints for the agents without paying any ex ante rents to them, while simultaneously providing higher-power incentives than under piece rates. The policy implication of our analysis is that firms should adopt relative performance evaluation over absolute performance evaluation regardless of whether the agents are liquidity (wealth) constrained or not.

Keywords Piece rates · Tournaments · Contests · Liquidity constraints

JEL Classification D82 · D21

1 Introduction

Even though linear contracts are only proxies of the theoretically optimal non-linear contracts, they are popular in several occupations or industries (e.g., sales, physician

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contracts with HMOs, contracts between processors and farmers, and faculty raises), partly because they are simple to design and easy to implement and enforce.[1] The most common linear contracts are the piece rate contract and the cardinal tournament. Both of these are applicable when cardinal data on individual agent performance are available. By contrast, ordinal (or rank-order) tournaments are informationally wasteful when data on the agents' cardinal performance are available (Holmström 1982). Under rank-order tournaments, the prizes are fixed ex ante. Under cardinal tournaments, the sum of prizes is fixed ex ante and relative performance evaluation determines each agent's share of the pie. Under the piece rate contract each agent is evaluated according to his absolute performance or according to his performance against a predetermined standard, while under the tournament each agent is evaluated relative to the performance of his peers. In particular, under both schemes each agent receives a base payment and a bonus payment, but the bonus payment is determined by absolute performance in piece rates and by relative performance in tournaments.[2] Following the footsteps of Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983), Malcomson (1984) and others, the comparison of cardinal tournaments and piece rates has been the subject of current literature (Tsoulouhas 1999; Tsoulouhas and Vukina 1999, 2001; Hueth and Ligon 2001; Wu and Roe 2005, 2006; Tsoulouhas and Marinakis 2007; Marinakis and Tsoulouhas 2009; Tsoulouhas 2010).[3] This comparison is important because it allows us to contrast the efficiency of absolute performance evaluation against relative performance evaluation.

Absent liquidity considerations, when agents are risk averse and production is subject to sufficiently large common shocks, the tournament is a superior incentive scheme to the piece rate. This is because the tournament uses the information generated by the performance of the group of participating agents as a whole, while the piece rate does not. Specifically, if the disturbance in the output of each agent is correlated with the disturbances in the output of the other agents, the information contained in the average production can be very useful to the principal in creating a contract which is a step closer to the First Best. Moreover, under the tournament, if the principal is risk-neutral or is less risk-averse than the agent, an insurer–insuree relationship can be developed between principal and agent allowing for a Pareto improvement of the contract. That is, the principal will offer insurance to the risk averse agent by filtering away the common shock from his responsibility. Insurance will make the agent more tolerant to a higher-power incentive scheme and, therefore, the agent is expected to increase his effort level.

[1] To some extent, the non-linearity of the theoretically optimal contract is due to the fact that contracts accommodate all possible events. Holmström and Milgrom (1987), however, have argued that schemes that adjust compensation to account for rare events may not provide correct incentives in ordinary high probability circumstances.

[2] The base payment ensures agent participation and the bonus provides incentives to perform. Under tournament an agent receives a bonus if his performance is above that of his peers, and a penalty otherwise.

[3] Note that Tsoulouhas (2010) examines a form of cardinal tournaments in which the weights on absolute and group average performance are not equal. Under these tournaments, the total prize is not fixed and the principal can increase his profit over standard tournaments, however, these tournaments are more difficult to design in practice and, hence, to the best of our knowledge, they are not used.

One might conjecture that the superiority of tournaments over piece rates may not survive under liquidity constraints. Tsoulouhas and Vukina (1999) and Marinakis and Tsoulouhas (2009) have proved that the optimality of tournaments over piece rates can break down when the risk-neutral principal is subject to a limited liability (bankruptcy) constraint, which limits the payments a principal can make, provided that the liquidation value of the principal's enterprise is sufficiently small. This is so because tournaments increase payments in unfavorable states, but these are the states in which the limited liability constraint comes into play. The intuition is that contracts with risk neutrality and limited liability for the principal look very much like those that would have been obtained with risk aversion. In other words, if the principal is concerned about the allocation of profit across states, he will no longer offer insurance against common shocks via tournaments and will resort to piece rate contracts or fixed performance standards. This paper investigates the optimality of tournaments over piece rates when the agent, instead, is subject to a liquidity constraint which introduces ex post limitations on the minimum payment the agent can accept or the maximum penalty that can be imposed on him (Innes 1990, 1993a,b). The liquidity constraint prevents the principal from compensating the agent by an amount smaller than a predetermined level in all states of nature.

The models used by Lazear and Rosen (1981), Green and Stokey (1983) and others allow the payments to the agents to be negative. In particular, under both the piece rate and the tournament payment schemes, if the agents produced a sufficiently low output they would usually have to pay the principal. Thus, according to the standard literature, if the production of an agent is sufficiently low the principal will penalize the agent by imposing a negative compensation and acquire whatever output the agent produced. This is certainly inconsistent with what we observe in reality.

The liquidity constraint is partly an institutional constraint on contracts. It is imposed by law for several industries in numerous countries. Such legislation aims at removing the burden of excessive penalties imposed on agents for negative outcomes beyond their control, rather than at maximizing social welfare. However, a liquidity constraint for the agent may alter the choice the principal makes between tournaments and absolute performance contracts. This can be due to a number of reasons. Some of these reasons are in favor of tournaments and some are in favor of piece rates. First, by increasing payments to the agents in unfavorable states, tournaments are more likely to satisfy tight liquidity constraints for the agents. Second, by providing insurance, tournaments may satisfy the liquidity constraints for the agents without paying rents to them. This is so because tournaments increase the compensation to the agents in unfavorable states but reduce the payments in favorable states. By contrast piece rates may pay the agents ex ante rents when the liquidity constraints are tight (i.e., when the minimum required payment to the agents is high), which reduces the principal's profit. If piece rates pay ex ante rents to the agents, they could be dominant over tournaments from the principal's perspective only if implemented effort under piece rates were sufficiently higher. But, in general, tournaments allow the principal to implement higher-power incentives than piece rates, which enhances the dominance of tournaments. Third, agents may be unable to pay for insurance especially in low states of nature if the liquidity constraints are tight, which works against tournaments. Fourth, the attitude of the principal and the agents toward risk may change. Liquidity constraints may make the agents more tolerant to risk, in the sense that if the agents know that their liability is limited, they may become indifferent among the range of states over which the liquidity constraint is binding.[4] On the other hand, the liquidity constraints for the agents are expected to make the principal care about the allocation of payments and, hence, profit across states to satisfy the liquidity constraints and ensure agent participation. When the principal becomes less tolerant to risk, while agents simultaneously become more tolerant to risk and, therefore, they are not willing to pay enough for insurance, the principal may find it less profitable to offer insurance to the agent through a tournament and may resort to piece rates again. Thus, in all, it is not a priori clear if tournaments, which are normally superior over piece rates when production is subject to common shocks, maintain their superiority under liquidity constraints for the agents.

Our analysis shows that, surprisingly, in the presence of sufficient common uncertainty a principal contracting with risk averse agents will prefer to offer a tournament even when agents are liquidity constrained. This finding is diametrically opposite to the result for the case when the principal, instead, is subject to limited liability. The rationale for this result follows directly from the discussion above. It turns out that by providing insurance against common shocks through a tournament, so that payments to the agents in unfavorable states increase and payments in favorable states decrease, the principal can satisfy tight liquidity constraints for the agents without paying any ex ante rents to them while simultaneously providing them with higher-power incentives than under piece rates. The individual rationality constraints for the agents are always binding under tournaments, whereas under piece rates they are non-binding (that is, the agents receive ex ante rents) when the liquidity constraints for the agents are really tight (that is, when the minimum payment required to satisfy the liquidity constraints is high). This finding establishes our claim that the principal can satisfy tight liquidity constraints for the agents without paying any ex ante rents to them under tournament. Our second claim, that the principal can implement higher-power incentives under tournament, is facilitated by the fact that the piece rate contract cannot be defined for a piece rate larger than one (in the sense that the principal would not make an offer such that marginal cost exceeded marginal revenue) whereas the tournament is defined for a larger bonus factor. The larger the minimum payment satisfying an agent's liquidity constraint, the higher the power of incentives the principal provides. In other words, the principal counterbalances the increase in the base payment, which is required to satisfy the liquidity constraint, with higher-power incentives in order to curb agent rents and in order to reduce the likelihood that output is low. Tournaments provide the principal with added flexibility in the determination of this power when the liquidity constraints are really tight.[5]

[4] This is certainly in accord with Laffont and Martimort (2002) who state (see p. 121): "A limited liability constraint on transfers implies higher-powered incentives for the agent. It is almost the same as what we would obtain by assuming that the agent is a *risk lover*. The limited liability constraint on transfers somewhat *convexifies* the agent's utility function."

[5] On the other hand, regardless of whether the principal offers a piece rate or a tournament, the liquidity constraints for the agents are non-binding (that is, in some sense, agents receive ex post rents) when the minimum payment required to satisfy the liquidity constraints is low. In that case, the analysis is similar

The empirical application that stems from our analysis is that firms should adopt relative performance evaluation via tournaments over absolute performance evaluation via piece rates regardless of whether the agents are liquidity (wealth) constrained or not. This finding enhances the generality of the results obtained in Lazear and Rosen (1981), Green and Stokey (1983) and Nalebuff and Stiglitz (1983). For instance, in the case of processor companies contracting with farmers who most often are liquidity constrained, processors need not fear that the farmers' liquidity issues detract from the superiority of tournaments.[6] Processors have been using tournaments for decades, especially with chicken growers, and there has been lots of mostly anecdotal evidence that farmers complain about such payment schemes in the presence of wealth constraints. Last but not least, note that similar to Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983) and Malcomson (1984) we are not looking for the optimal contract, instead, we contrast the efficiency properties of absolute to relative performance evaluation by comparing cardinal tournaments to piece rates the way they are used in practice.[7]

Section 2 presents our model and Sect. 3 presents the benchmark case without liquidity constraints with mostly known results in the literature. Section 4 presents our results when the agents are liquidity constrained and Sect. 5 determines the dominant compensation scheme. Section 6 extends the analysis by relaxing the assumption of risk-aversion for the agents. Section 7 concludes. Appendix A presents the main results of a representative numerical analysis. Appendix B presents the results of an analysis if common uncertainty were relatively low.

2 Model

A principal signs a contract with n homogeneous agents.[8] Each agent i produces output according to the production function $x_i = a + e_i + \eta + \varepsilon_i$, where a is the agent's known ability, e_i is his effort, η is a common shock and ε_i is an idiosyncratic shock. The idiosyncratic shocks, ε_i, and the common shock follow independent

Footnote 5 continued
to the benchmark case in Lazear and Rosen (1981), Green and Stokey (1983) and Nalebuff and Stiglitz (1983), and tournaments are optimal under sufficient common uncertainty.

6 Wealth constraints can certainly be a concern in contracts for salesmen as well.

7 Kim (1997) analyzes a setting with a risk neutral principal and a single risk neutral agent whose liability is limited. He shows that the optimal contract is a bonus contract in which the principal and the agent share the output, and the agent receives an additional fixed bonus only when output is greater than some predetermined level. We differ from Kim because in our model there are multiple agents who are risk-averse and their activities are subject to common uncertainty. Also see footnote 12 below.

8 Agent heterogeneity has been examined in a number of recent papers. In a model similar to ours, Theilen (2003) examines optimal piece rate contracts in the presence of moral hazard and adverse selection. Tsoulouhas et al. (2007) consider CEO contests that are open to heterogeneous outsider contestants. Mathews and Namoro (2008) examine the entry choice of heterogeneous agents over tournaments with different prize levels. Konrad and Kovenock (2010) examine discriminating contests with stochastic contestant abilities. Kolmar and Sisak (2007) analyze discriminating contests among heterogeneous contestants. Tsoulouhas and Marinakis (2007) analyze ex post agent heterogeneity to make the point that agent heterogeneity compromises the insurance function of tournaments. Instead, Riis (2010) allows for agents who are heterogeneous ex ante.

distributions. Each agent's effort and the subsequent realizations of the shocks are private information to him, but the output obtained is publicly observed. The principal compensates agents for their effort based on their outputs by using a piece rate contract or a cardinal tournament. Agent preferences are represented by a CARA utility function $u(w_i, e_i) = -\exp\left(-w_i + \frac{e_i^2}{2a}\right)$, where the agent's coefficient of absolute risk aversion is set equal to 1 for simplicity. The cost of effort is measured in monetary units. Each agent has a reservation utility $-\exp(-\overline{u})$, where $\overline{u}$ is arbitrary (zero or positive).

3 Piece rates and tournaments without liquidity constraints

We start by deriving the optimal contractual variables for the piece rate and the tournament without liquidity constraints for the agents. We assume that the total production disturbance, $\eta + \varepsilon_i$, follows a normal distribution with zero mean and variance equal to $c/\sqrt{2\pi}$, and the idiosyncratic shock, ε_i, follows a normal distribution with zero mean and variance equal to $d/\sqrt{2\pi}$.[9]

3.1 Piece rates

The piece rate contract (R) is the payment scheme in which the compensation to the i^{th} agent is $w_i = b_R + \beta_R x_i$, where (b_R, β_R) are the contractual variables to be determined by the principal. The principal determines the optimal values of these parameters by backward induction. Thus, the principal calculates each agent's expected utility

$$EU_R = -\exp\left(-b_R - \beta_R(a + e_i) + \frac{e_i^2}{2a} + \frac{\beta_R^2 c}{2\sqrt{2\pi}}\right). \tag{1}$$

To ensure the compatibility of the contract with agent incentives to perform, the principal calculates the effort level that maximizes (1). First order conditions yield[10]

$$e_i = a\beta_R. \tag{2}$$

To ensure the compatibility of the contract with agent incentives to participate, the principal selects the value of the base payment, b_R, that satisfies the agent's individual rationality constraint with equality so that the agent receives no rents but still accepts the contract. The agent's individual rationality constraint satisfies $EU_R = -\exp(-\overline{u})$, where EU_R is determined by (1) and (2). Solving for b_R implies

$$b_R = \overline{u} + \frac{\frac{c}{\sqrt{2\pi}} - a}{2}\beta_R^2 - a\beta_R. \tag{3}$$

[9] As will become obvious in the remaining analysis, this assumption on the variance simplifies the exposition.

[10] Note that the concavity of the utility function implies that first order conditions are sufficient.

Thus, by choosing the piece rate β_R, the principal can precisely determine the agent's effort because the agent will optimally set his effort according to (2). In addition, by setting b_R in accordance with (3) the principal can induce agent participation at least cost. That is, agent incentives to perform are only determined by the piece rate β_R, whereas agent incentives to participate are determined by the base payment b_R.

Given conditions (2) and (3) the principal maximizes his expected total profit

$$ET\Pi_R = \sum_{i=1}^{n} [Ex_i - Ew_i] = n \left[a + a\beta_R - \frac{\frac{c}{\sqrt{2\pi}} + a}{2} \beta_R^2 - \overline{u} \right]. \tag{4}$$

The solution to this problem satisfies

$$\beta_R = \frac{a}{a + \frac{c}{\sqrt{2\pi}}}. \tag{5}$$

Condition (3) then implies

$$b_R = \overline{u} - \frac{a^2}{2} \frac{\frac{c}{\sqrt{2\pi}} + 3a}{\left[\frac{c}{\sqrt{2\pi}} + a\right]^2}. \tag{6}$$

Given conditions (5) and (4) expected profit per agent is

$$E\Pi_R = a + \frac{1}{2} \frac{a^2}{a + \frac{c}{\sqrt{2\pi}}} - \overline{u}. \tag{7}$$

3.2 Tournaments

The cardinal tournament (T) is the payment scheme in which the compensation to each agent is determined by a relative performance evaluation. Specifically, $w_i = b_T + \beta_T(x_i - \overline{x})$, where $\overline{x}$ is the average output obtained by all agents and (b_T, β_T) are the contractual variables to be determined by the principal.[11] Under a tournament the agent's expected utility is

$$EU_T = -\exp\left(-b_T - \beta_T \frac{n-1}{n}(a + e_i) + \beta_T \frac{1}{n} \Sigma_{j \neq i}(a + e_j) + \frac{e_i^2}{2a} + \frac{1}{2} \frac{n-1}{n} \frac{\beta_T^2 d}{\sqrt{2\pi}}\right). \tag{8}$$

[11] Again, as mentioned earlier, "hybrid" cardinal tournaments of the form $w_i = b_T + \beta_T x_i - \gamma_T \overline{x}$ are analyzed in Tsoulouhas (2010). These tournaments are more difficult to design in practice and, to the best of our knowledge, they are not used.

The effort level that maximizes (8) satisfies

$$e_i = \frac{n-1}{n} a\beta_T. \tag{9}$$

Further, the individual rationality constraint $EU_T = -\exp(-\overline{u})$ implies

$$b_T = \overline{u} + \frac{1}{2}\frac{n-1}{n}\left(\frac{n-1}{n}a + \frac{d}{\sqrt{2\pi}}\right)\beta_T^2. \tag{10}$$

Then, given conditions (9) and (10), the principal maximizes expected total profit

$$ET\Pi_T = n\left[a + \frac{n-1}{n}a\beta_T - \frac{1}{2}\frac{n-1}{n}\left(\frac{n-1}{n}a + \frac{d}{\sqrt{2\pi}}\right)\beta_T^2 - \overline{u}\right]. \tag{11}$$

The solution to the principal's maximization problem satisfies

$$\beta_T = \frac{a}{\frac{n-1}{n}a + \frac{d}{\sqrt{2\pi}}}, \tag{12}$$

therefore,

$$b_T = \overline{u} + \frac{1}{2}\frac{a^2}{a + \frac{n}{n-1}\frac{d}{\sqrt{2\pi}}}. \tag{13}$$

Given (12) and (11) expected profit per agent is

$$E\Pi_T = a + \frac{1}{2}\frac{a^2}{a + \frac{n}{n-1}\frac{d}{\sqrt{2\pi}}} - \overline{u}. \tag{14}$$

By comparing (4) to (14) it can easily be shown that

$$E\Pi_T > E\Pi_R \Leftrightarrow \frac{n}{n-1}d < c, \tag{15}$$

that is, tournaments are superior when total uncertainty is large relative to the idiosyncratic uncertainty (equivalently, when common uncertainty is relatively large) and when the number of agents is large. This is so because tournaments eliminate common uncertainty but they add the average individual noise of others. It is also straightforward to show that $b_T > b_R$ and $\beta_T > \beta_R$. The rationale behind the first inequality is that the expected bonus payment under tournament is zero, whereas that under piece rate is positive. Therefore, agents expect to be compensated for effort through the base payment in a tournament. The intuition behind the second inequality is that the principal implements higher-power incentives when common uncertainty is removed from the responsibility of the agent under tournament. By comparing effort under piece rates in (2) to that under tournaments in (9), it follows that effort under tournaments is larger provided that common uncertainty is sufficiently large, that is, $[n/(n-1)]d < c$, which is a prerequisite for tournaments to be superior to begin with. Specifically,

$$a\beta_R < \frac{n-1}{n} a\beta_T \Leftrightarrow \frac{n}{n-1} d < c.$$

4 Piece rates and tournaments with liquidity constraints

Next we turn to the case with liquidity constraints for the agents. The liquidity constraint is

$$w_i \geq \underline{w}, \tag{16}$$

where $\underline{w}$ is the minimum permissible payment. The liquidity constraints for the agents necessitate a support for the production shocks which is bounded below and above. The support must be bounded below so that in the worst possible output state the liquidity constraints are still satisfied (obviously they cannot be satisfied with an output space which is unbounded below). For a similar reason, the support must be bounded above to eliminate the case when the payment under tournament is below the minimum required to satisfy the liquidity constraint when average output is unbounded above.[12] With bounded support for the production shocks one might expect that the First Best is always implementable by punishing the agent severely for outcomes outside the support (see p. 140 in Bolton and Dewatripont 2005). Note, however, that the liquidity constraints of the agents prevent severe punishment of them.

The requirement of bounded support eliminates unbounded distributions such as the normal, which we used in Sect. 3 (and is typically used in the literature for the setting without liquidity constraints). The normal distribution is one of the assumptions that are made in order to obtain a closed form solution for the case without liquidity constraints. Further, a truncated normal distribution provides neither a closed form solution nor a numerical one. However, we were able to obtain significant insight through a numerical analysis by assuming that the idiosyncratic and the common shocks follow independent uniform distributions, in which case the sum of these shocks follows a triangular distribution. Specifically, the idiosyncratic shocks, ε_i, follow independent uniform distributions with support $[-d, d]$, and therefore, the total production shock, $v_i \equiv \varepsilon_i + \eta$, follows a triangular distribution with density $f(\cdot)$, the support of which is

[12] An alternative approach would be to consider a modification of the payment schemes such that the agent still receives the minimum payment required to satisfy his liquidity constraint, $\underline{w}$, specifically, consider $\max\{\underline{w}, w_i\}$ where w_i is determined by the scheme. We ignore this approach for two reasons. First, this paper compares simple linear cardinal tournaments to piece rates the way they are used in the empirical applications stated earlier. Kinked schemes of the form above are not used in these applications. Second, we pursued the analysis of the kinked case and it was intractable because the expected utility of a non-linear (concave) transformation of a kinked payment scheme cannot be calculated analytically. However, we determined that the results are qualitatively similar.

assumed to be $[-c, c]$ with zero mean.[13] The following lemmata apply to piece rates and tournaments with liquidity constraints.

Lemma 1 *Under piece rates, when the agents are subject to liquidity constraints in addition to individual rationality constraints, at least one of the individual rationality and the liquidity constraints for each agent binds depending on the values of parameters $\underline{w}, \overline{u}, a$ and c.*

Proof First note that if both constraints were non-binding, then, the principal would reduce the payments to the agent until one of the two constraints became binding (that is, until the agent received no rents in an ex ante or in an ex post sense). As shown in Sect. 3, solving without the liquidity constraint for each agent (in which case the individual rationality constraint is obviously binding) implies that the contractual variables (b_R, β_R) satisfy conditions (6) and (5), respectively, and therefore the payment w_i may or may not satisfy the liquidity constraint in all states depending on the values of parameters $\underline{w}, \overline{u}, a$ and c. Therefore, when the individual rationality constraint is binding, the liquidity constraint is binding or non-binding (the latter when $\underline{w}$ is relatively low). Solving without the individual rationality constraint (in which case the liquidity constraint is obviously binding in the lowest possible state) implies that the payments to the agent may or may not satisfy the individual rationality constraint depending on the values of parameters $\underline{w}, \overline{u}, a$ and c again. Therefore, when the liquidity constraint is binding the individual rationality constraint is binding or non-binding (the latter when $\underline{w}$ is relatively large). □

Lemma 2 *Under tournaments, when the agents are subject to liquidity constraints in addition to individual rationality constraints, and assuming that the regularity condition $a > [n/(n-1)]d$ holds, the individual rationality constraint for each agent is always binding and the liquidity constraint for each agent is binding or non-binding depending on the values of parameters $\underline{w}, \overline{u}, a$ and d.*

Proof First, similar to Lemma 1, the two constraints cannot simultaneously be non-binding. Solving without the individual rationality constraint (in which case the liquidity constraint is obviously binding in the lowest possible state) implies that $b_T = \underline{w} + \beta_T d$. This is so because $w_i = b_T + \beta_T (x_i - \overline{x}) = \underline{w}$ and, given that $\varepsilon_i \in [-d, d]$, if the number of agents is sufficiently large $x_i - \overline{x} \overset{D}{\to} uniform[-d, d]$. Then, since the principal's profit per agent is $\Pi_i = x_i - b_T = a - \underline{w} + \left(\frac{n-1}{n} a - d\right) \beta_T + \eta + \varepsilon_i$, it follows that expected profit per agent is $E\Pi_T = a - \underline{w} + \left(\frac{n-1}{n} a - d\right) \beta_T$. To maximize this expected profit the principal chooses the maximum β_T that satisfies the individual rationality constraint with equality so that the agent accepts the contract. Therefore, the individual rationality constraint is always binding. As shown in Sect. 3, solving

[13] Note that the triangular distribution we use in our numerical analysis with liquidity constraints is a first-order approximation of the normal distribution we use in the analysis without liquidity constraints. Again, the model without the liquidity constraint can be analyzed and provide a closed form solution when the total shock follows a normal distribution, however, it does not provide even a numerical solution when it follows a triangular distribution. On the other hand, the model with the liquidity constraint can provide a numerical solution with a triangular distribution, but it does not provide even a numerical solution with a normal distribution.

without the liquidity constraint (in which case the individual rationality constraint is obviously binding) implies that the contractual variables $(b_{T,} \beta_T)$ satisfy (13) and (12), respectively, and therefore the payment w_i may or may not satisfy the liquidity constraint in all states depending on the values of parameters $\underline{w}$, $\overline{u}$, a and d. Therefore, when the individual rationality constraint is binding, the liquidity constraint is binding or non-binding (the latter when $\underline{w}$ is relatively low). □

Note that the regularity condition $a > [n/(n-1)]d$ requires that agents are of sufficiently high ability. The proof of Lemma 2, then, shows that the principal's profit is increasing in the bonus factor $\beta_{T.}$ The rationale why the individual rationality constraint is always binding for the tournament case but not for the piece rate case is that profit is decreasing in the piece rate β_R. Therefore, unlike the tournament case in which the principal benefits by increasing the bonus factor β_T until it yields no rents to the agent, in the piece rate case the principal may prefer to provide the agent with rents in order to increase his profit. Thus, there is a fundamental difference between tournaments and piece rates in this respect, which drives the results in our paper.

4.1 Piece rates

We start by analyzing the piece rate case. The piece rate scheme can be written as $w_i = b_R + \beta_R (a + e_i + v_i)$. As Lemma 1 indicates, the individual rationality constraint can be binding or not. Because of this, the procedure for determining the contractual variables is somewhat different than the one we followed above for the case without liquidity constraints (without liquidity constraints the individual rationality constraints are always binding). With liquidity constraints, we determine the base payment b_R through these constraints, and the piece rate β_R from the profit maximizing condition. Then we check whether this solution satisfies the individual rationality constraints.

Clearly, if the payment satisfies the liquidity constraint (16) in the lowest possible state, then, it satisfies the constraint in all states because the payment scheme is increasing in the state. Therefore, if the constraint is binding in the lowest state, then it is non-binding in all states. From the agent's perspective, given that the principal controls incentives through the payment scheme, the worst state is the one in which the principal provides him no incentives to perform and the production state turns out to be the worst, that is, $e_i = 0$ and $v_i = -c$.[14] In the remaining analysis we focus on the case when the liquidity constraint is binding in the lowest possible state. Therefore, the principal will set

[14] Recall that the liquidity constraint is an institutional constraint which prohibits penalizing the agent for obtaining a low output, and it should hold regardless of whether the contract is optimal or not. The agent's optimal response under the contract should not be included in the calculation of the required base wage, because we cannot assume the optimal contract in setting up the constraint. Instead, the constraint determines the agent's optimal response under contract and the optimal contract. By setting effort at the lowest possible level, and by setting the state at the worst possible realization, we can ensure that the piece rate contract always satisfies the agent's liquidity constraint. Note that under tournaments, instead, and given the homogeneity of agents, efforts cancel out of the payment calculation.

$$b_R = \underline{w} - \beta_R (a - c). \tag{17}$$

The expected utility for the agent is

$$EU_i = -\int_{-c}^{c} \exp(-\beta_R v_i) f(v_i) dv_i \exp\left(-\underline{w} - \beta_R c - \beta_R e_i + \frac{e_i^2}{2a}\right). \tag{18}$$

To provide correct incentives to the agent, the principal calculates the effort level e_i that maximizes (18). First order conditions yield

$$-\int_{-c}^{c} \exp(-\beta_R v_i) f(v_i) dv_i \exp\left(-\underline{w} - \beta_R c - \beta_R e_i + \frac{e_i^2}{2a}\right)\left(-\beta_R + \frac{e_i}{a}\right) = 0$$

and, because neither $\int_{-c}^{c} \exp(-\beta_R v_i) f(v_i) dv_i$ nor $\exp\left(-\underline{w} - \beta_R c - \beta_R e_i + \frac{e_i^2}{2a}\right)$ can be equal to zero, it follows that

$$e_i = a\beta_R. \tag{19}$$

The principal's profit per agent is $\Pi_i = (1 - \beta_R) x_i - b_R = a + a\beta_R - a\beta_R^2 - \underline{w} - \beta_R c + (1 - \beta_R) v_i$. Then the expected profit per agent is

$$E\Pi_R = a + a\beta_R - a\beta_R^2 - \underline{w} - \beta_R c. \tag{20}$$

Maximizing the expected profit with respect to β_R yields

$$\beta_R = \frac{a - c}{2a}. \tag{21}$$

Hence, given the contractual variables and the optimal effort level for the agent, the expected profit per agent is

$$E\Pi_R = \frac{5}{4}a + \frac{1}{4}\frac{c^2}{a} - \frac{1}{2}c - \underline{w}. \tag{22}$$

Note that condition (22) indicates that the principal will make an offer only if $\underline{w}$ is relatively low, otherwise production is unprofitable.

Given conditions (17) and (21), the individual rationality constraint requires

$$-\int_{-c}^{c} \exp\left(-\frac{a - c}{2a} v_i\right) f(v_i) dv_i \exp\left(-\underline{w} - \frac{3}{4}c + \frac{5}{8}\frac{c^2}{a} + \frac{1}{8}a\right) \geq -\exp(-\overline{u}). \tag{23}$$

Clearly, (23) may or may not hold, depending on the values of parameters $\underline{w}$, $\overline{u}$, a and c. If it holds, then the contractual variables to be offered by the principal satisfy (17) and (21). If (23) does not hold, that is, if β_R in (21) violates the individual rationality constraint, then the individual rationality constraint is binding. In this case, β_R must be determined through the individual rationality constraint with equality. Given (18), (19) and the density function for v_i, the individual rationality constraint is written as

$$\int_{-c}^{c} \exp\left(-\beta_R v_i\right) \frac{c - |v_i|}{c^2} dv_i \exp\left(\frac{1}{2} a\beta_R^2 - \beta_R c\right) \exp(-\underline{w}) = \exp(-\overline{u}). \qquad (24)$$

Given that $c > 0$, (24) is equivalent to

$$\frac{\left[1 + \exp\left(2\beta_R c\right) - 2\exp\left(\beta_R c\right)\right] \exp\left(-\beta_R c\right)}{\beta_R^2 c^2} \times \exp\left(\frac{1}{2} a\beta_R^2 - \beta_R c\right) \exp(\overline{u} - \underline{w}) - 1 = 0. \qquad (25)$$

A closed form solution for β_R is impossible to obtain from (25). As a result we have to rely on computational methods in order to determine the piece rate values β_R which are individually rational. Our computations proceed as follows: we derive the contractual variables from Eqs. (17) and (21) assuming that the liquidity constraint is binding in the lowest state and ignoring the individual rationality constraint. Then we check if the individual rationality constraint (23) is satisfied by the solution (in which case it is non-binding) or if it is violated (in which case it is binding). If (23) is found to be binding, then the piece rate β_R is derived by the solution of (25) using a Newton algorithm and b_R is still determined by (17). In this case, when we have multiple solutions for β_R, we keep the one maximizing the principal's profit. If (23) is found to be non-binding we keep the solutions from Eqs. (17) and (21).

4.2 Tournaments

Next, we turn to the tournament case. Recall that under the tournament the compensation to each agent is $w_i = b_T + \beta_T \left(x_i - \overline{x}\right)$, which can be written as $w_i = b_T + \beta_T \left(e_i - \overline{e}\right) + \beta_T \vartheta_i$, where $\vartheta_i \equiv \varepsilon_i - \overline{\varepsilon}$, with $\overline{e}$ denoting the average effort and $\overline{\varepsilon}$ denoting the average idiosyncratic shock. Given that the agents are homogeneous, the contract is uniform for all agents and the optimal effort level is equal in equilibrium for all agents.[15] Thus, the compensation to each agent can be expressed as $w_i = b_T + \beta_T \vartheta_i$. As shown in the proof of Lemma 2,

15 Skaperdas and Gan (1995) have shown that if the agents are heterogeneous in their risk-aversion rates, then, their equilibrium effort can be sensitive to the risk-aversion rate. In a model with limited liability for the agents, they show that the more risk-averse agent exerts higher effort as insurance against loss. Instead of heterogeneous agents, Konrad and Schlesinger (1997) assume that agents are identical and examine the effects of risk-aversion on rent-seeking and rent-augmenting expenditures to show that risk-aversion may have ambiguous effects on expenditures in rent-seeking contests. Our analysis indicates that our results are qualitatively correct provided that the agents are risk-averse, but they depend quantitatively on the risk-aversion rate.

$$b_T = \underline{w} + \beta_T d. \tag{26}$$

Similar to piece rates, if the liquidity constraint is binding in the lowest state, then it is non-binding in all states, because the payment under tournament is also increasing in the state. The agent's expected utility is

$$\begin{aligned} EU_i &= -\int_{-d}^{d} \exp\left(-\beta_T \vartheta_i\right) f(\vartheta_i) d\vartheta_i \\ &\quad \times \exp\left(-\underline{w} - \beta_T d - \beta_T \frac{n-1}{n} e_i + \beta_T \frac{1}{n} \Sigma_{\substack{j=1 \\ j \neq i}}^{n} e_j + \frac{e_i^2}{2a}\right). \end{aligned} \tag{27}$$

The effort level that maximizes the agent's expected utility satisfies

$$\begin{aligned} &-\int_{-d}^{d} \exp\left(-\beta_T \vartheta_i\right) f(\vartheta_i) d\vartheta_i \exp\left(-\underline{w} - \beta_T d - \beta_T \frac{n-1}{n} e_i + \beta_T \frac{1}{n} \Sigma_{\substack{j=1 \\ j \neq i}}^{n} e_j + \frac{e_i^2}{2a}\right) \\ &\quad \times \left(-\beta_T \frac{n-1}{n} + \frac{e_i}{a}\right) = 0. \end{aligned}$$

Because the product of the first two terms in the equation cannot be equal to zero, it follows that

$$e_i = \frac{n-1}{n} a \beta_T. \tag{28}$$

Given Lemma 2, which states that the individual rationality constraint is always binding under tournaments with liquidity constraints, the principal chooses the value of the bonus factor β_T that satisfies the agent's individual rationality constraint with equality. Thus, (26), (27) and (28) imply

$$\begin{aligned} EU_i &= -\int_{-d}^{d} \exp\left(-\beta_T \vartheta_i\right) f(\vartheta_i) d\vartheta_i \exp\left(-\underline{w} - \beta_T d + \frac{1}{2}\left(\frac{n-1}{n}\right)^2 a \beta_T^2\right) \\ &= -\exp(-\overline{u}). \end{aligned} \tag{29}$$

Note that in equilibrium the random variable $x_i - \overline{x} \equiv \vartheta_i \xrightarrow{D} \textit{uniform}[-d, d]$.[16] Hence, $\int_{-d}^{d} \exp(-\beta_T \vartheta_i) f(\vartheta_i) d\vartheta_i$ converges to

$$\int_{-d}^{d} \exp(-\beta_T \vartheta_i) \frac{1}{2d} d\vartheta_i = \frac{\exp(\beta_T d) - \exp(-\beta_T d)}{2\beta_T d}.$$

Then, (29) becomes

$$-\frac{\exp(\beta_T d) - \exp(-\beta_T d)}{2\beta_T d} \exp\left(-\underline{w} - \beta_T d + \frac{1}{2}\left(\frac{n-1}{n}\right)^2 a\beta_T^2\right) = -\exp(-\overline{u})$$

$$\Leftrightarrow \exp\left(\frac{1}{2}\left(\frac{n-1}{n}\right)^2 a\beta_T^2 - \beta_T d\right) \exp(\overline{u} - \underline{w}) = \frac{2\beta_T d}{\exp(\beta_T d) - \exp(-\beta_T d)}. \tag{30}$$

Clearly, similar to the piece rate case, Eq. (30) has no closed form solution. A solution can only be obtained by computational methods (recall that we use a Newton algorithm). The principal's profit per agent is $\Pi_i = x_i - b_T = a - \underline{w} + \left(\frac{n-1}{n} a - d\right) \beta_T + \varepsilon_i + \eta$. Hence, given the optimal base payment and the optimal effort level for the agent, the expected profit per agent is

$$E\Pi_T = a - \underline{w} + \left(\frac{n-1}{n} a - d\right) \beta_T. \tag{31}$$

where β_T can only be determined numerically by solving (30).

5 The dominant contract under liquidity constraints

The principal's decision about which compensation scheme to offer depends entirely on expected profits. Clearly, under both schemes, expected profits decline when a liquidity constraint is introduced in addition to the other constraints. Our analysis indicates that these profits decline faster under piece rates as the liquidity constraint becomes tighter. The intuition behind our result is that the liquidity constraint distorts the agent's incentives to perform because it reduces the penalty the principal can impose for unfavorable outcomes. Therefore, the principal needs to provide higher-power incentives. By filtering common shocks from the responsibility of the agent, tournaments make the agent more tolerant to higher-power incentives, hence, it is easier for the principal to implement higher-power incentives under tournament than under piece rates. Moreover, the piece rate β_R cannot exceed 1 (i.e., because marginal

[16] Convergence in distribution utilizes the Central Limit Theorem, which involves the limit of the distribution's cdf as n converges to infinity. For n around 30 and above, the convergence becomes near absolute. For a value of $n = 100$ (which is used in Appendix A) the convergence error becomes smaller than machine precision (2.21e-016).

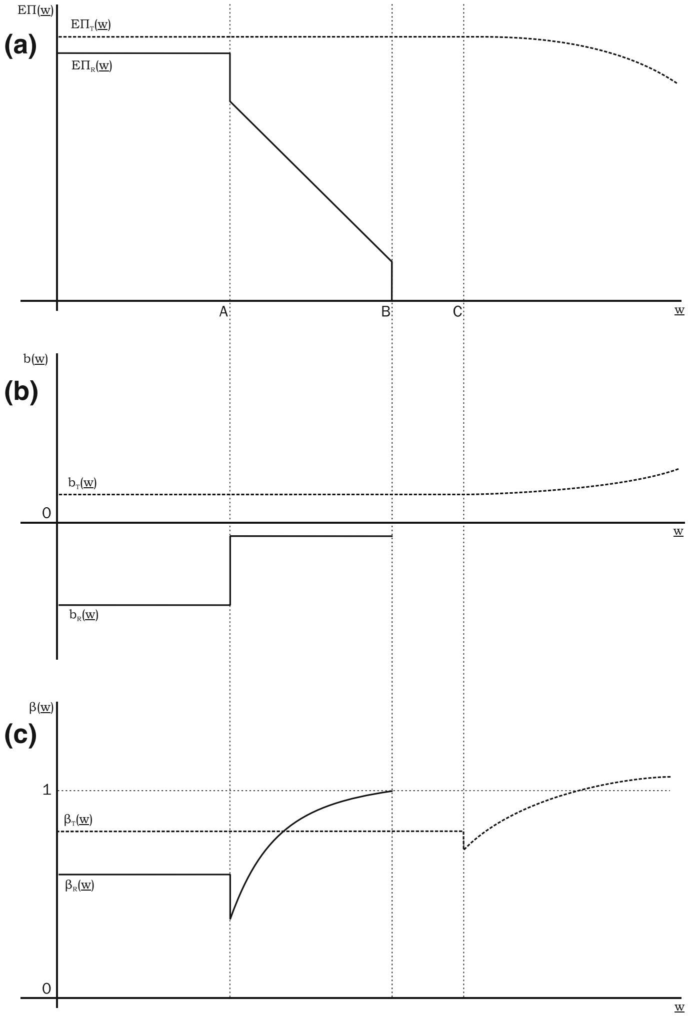

Fig. 1 The expected profit per agent and the contractual variables for the piece rate contract and the tournament

cost cannot exceed marginal revenue). By contrast, the bonus factor β_T can exceed 1 which enables the implementation of higher-power incentives.

Figure 1 illustrates that tournaments are dominant over piece rates when liquidity constraints are introduced. In particular, panel (a) shows that expected profit is always strictly larger under tournament regardless of the value of $\underline{w}$, that is, regardless of how tight the liquidity constraint is. Note that in our numerical analysis (see Appendix A), the results of which are depicted in Fig. 1, we assume that $[n/(n-1)]d < c$. That is, we assume that common uncertainty is sufficiently large relative to the idiosyncratic uncertainty. As shown in Appendix A, we use the parameter set $a = 10$, $n = 100$, $c = 3$, $d = 0.5$, however, the results are qualitatively the same for any parameters satisfying the regularity conditions $[n/(n-1)]d < c$ and $[n/(n-1)]d < a$. For the case without liquidity constraints expected profits per agent are calculated by using conditions (7) and (14). For the case with binding liquidity constraints expected profits per agent are calculated by using condition (20) where β_R is determined either by (21) or by the numerical solution of (25), and condition (31) where, again, β_T is determined numerically by (30). Obviously, for the range over which the liquidity constraint is non-binding, expected profit is flat and independent of $\underline{w}$. We confirmed this result for all possible values of common uncertainty that satisfy condition (15). A sufficient increase in the minimum permissible wage $\underline{w}$ decreases the expected profit under both schemes, but it does so much faster under piece rates. In fact, piece rates cannot be defined at all after a critical value of $\underline{w}$ is passed (see point B in panel (a)), because the principal needs to offer a piece rate larger than 1 to provide correct incentives to the agent. However, given that β_R cannot exceed 1, piece rates cannot be defined.[17] In interpreting the results depicted in Fig. 1, note that for $\underline{w}$ in the range up to A the individual rationality constraint under piece rates is binding and the liquidity constraint is non-binding. For $\underline{w}$ in the AB range the individual rationality constraint under piece rates is binding or non-binding and the liquidity constraint is binding. Under tournaments, the individual rationality constraint is always binding (see Lemma 2). Lastly, for $\underline{w}$ in the range up to C the limited liability constraint under tournament is non-binding.

Again, in our numerical analysis we assume that common uncertainty is relatively large, that is, $[n/(n-1)]d < c$. If, instead, common uncertainty were relatively low, that is, if $[n/(n-1)]d > c$, absent a liquidity constraint for the agent the tournament would be suboptimal, as condition (15) indicates. In terms of adjustments to panel (a) in Fig. 1, expected profit under piece rates would be higher up to point A. Recall that the piece rate is not defined beyond point B. Further, over range AB, expected profit under tournaments is shown by condition (14), and expected profit under piece rates is shown by condition (22) if the individual rationality constraint under piece rates is non-binding. Thus, expected profit under tournament would be larger if

$$\frac{1}{2}\left(\frac{a^2}{a+\frac{n}{n-1}\frac{d}{\sqrt{2\pi}}}+c\right)-\frac{1}{4}\left(a+\frac{c^2}{a}\right)>\overline{u}-\underline{w}.$$

Hence, if $\overline{u}-\underline{w}$ is sufficiently small, the principal will still find it optimal to switch to a tournament even though common uncertainty is relatively low, primarily for the sake of satisfying the liquidity constraint. That is, the liquidity constraint can reverse the preferences of the principal. Appendix B provides the numerical analysis when the individual rationality constraint under piece rates is binding, instead. This analysis also supports the finding above.[18]

Panel (b) indicates that the base payment is always larger (and positive) under tournament, but it increases with $\underline{w}$, that is, when the minimum acceptable payment

[17] In other words, the principal does not find it profitable to make an offer that the agent will accept.

[18] We are thankful to an anonymous referee for highlighting the importance of this possibility for the analysis.

increases the base payment must also increase to provide correct incentives to the agent to participate. As mentioned earlier, the rationale behind the positive b_T and the negative b_R is that the expected bonus payment under tournament is zero, whereas that under piece rate is positive. Therefore, agents expect to be compensated for effort through the base payment in a tournament. Under piece rates the bonus also compensates the agents for their effort costs. In fact, the expected bonus exceeds the cost of effort and the base payment is negative. Further, panel (c) indicates that both the piece rate β_R and the bonus factor β_T increase when $\underline{w}$ increases and the liquidity constraint is binding. There are two reasons for this: First, because the base payment increases when $\underline{w}$ increases, the principal must provide the agents with higher-power incentives in order to exert more effort and make up in lost profit due to the increase in the base payment. Second, when $\underline{w}$ increases, the principal provides the agents with higher-power incentives in order to minimize the likelihood that output is low and the principal is forced by the liquidity constraint to pay the minimum acceptable wage to the agent when, absent the constraint, it would have been optimal to pay less or impose a penalty. Again, note that piece rates are not defined for a piece rate above 1, whereas under tournaments the principal can continue to provide incentives to the agents through a bonus factor larger than 1, which explains the increased dominance of tournaments over piece rates for large values of the minimum acceptable payment $\underline{w}$.

6 Piece rates and tournaments with risk-neutrality

We now simplify our model by assuming that the agents are risk-neutral. Relaxing risk-aversion allows us to obtain closed form solutions for the contractual parameters even with liquidity constraints. Under risk-neutrality and without liquidity constraints, the determination of the optimal bonus factor for both the piece rate and the tournament leads to trivial results, especially because the provision of insurance is a major issue in contrasting piece rates to tournaments. This is the reason why our baseline model includes risk-aversion for the agents, even though a closed form solution cannot be obtained. The analysis with risk-neutrality and liquidity constraints, however, conforms with our findings for the more interesting risk-averse case. Agent preferences are represented by $u(w_i, e_i) = w_i - \frac{e_i^2}{2a}$. We assume that both production shocks η and ε_i follow identical and independent distributions with zero means. The support of $\eta + \varepsilon_i$ is $[-c, c]$ and the support of ε_i is $[-d, d]$. We start with the case of no liquidity constraints for the agents.

Under a piece rate contract $w_i = b_R + \beta_R x_i$, the agent's utility is

$$u(w_i, e_i) = b_R + \beta_R e_i + \beta_R a + \beta_R \eta + \beta_R \varepsilon_i - \frac{e_i^2}{2a},$$

hence, his expected utility is

$$EU_R = b_R + \beta_R e_i + \beta_R a - \frac{e_i^2}{2a}.$$

Maximizing expected utility with respect to e_i yields a condition identical to (2) above. The agent's individual rationality constraint then implies

$$b_R = \overline{u} - \frac{a}{2}\beta_R^2 - a\beta_R.$$

The principal's expected profit per agent is

$$E\Pi_R = a + a\beta_R - \frac{1}{2}a\beta_R^2 - \overline{u}.$$

Maximizing with respect to β_R yields

$$\beta_R = 1.$$

The intuition is that when agents are risk-averse, the optimal choice under piece rates is the solution of "selling the enterprise to the agent" (i.e., a piece rate of 1 so that, in light of moral hazard, the agent assumes all the risk). It follows that

$$b_R = \overline{u} - \frac{3}{2}a,$$

and expected profit per agent at the optimum piece rate is

$$E\Pi_R = \frac{3}{2}a - \overline{u}.$$

Under a tournament $w_i = b_T + \beta_T\,(x_i - \overline{x})$, the agent's utility is:

$$u\,(w_i, e_i) = b_T + \beta_T\left(\frac{n-1}{n}\,(e_i + \varepsilon_i) - \frac{1}{n}\sum_{j\neq i}\left(e_j + \varepsilon_j\right)\right) - \frac{e_i^2}{2a},$$

hence, his expected utility is

$$EU_T = b_T + \beta_T\left(\frac{n-1}{n}e_i - \frac{1}{n}\sum_{j\neq i}e_j\right) - \frac{e_i^2}{2a}.$$

Maximizing expected utility with respect to e_i yields a condition identical to (9) above. The agent's individual rationality constraint then implies

$$b_T = \overline{u} + \frac{1}{2}\left(\frac{n-1}{n}\right)^2 a\beta_T^2.$$

The principal's expected profit per agent is

$$E\Pi_T = a + \frac{n-1}{n}a\beta_T - \frac{1}{2}\left(\frac{n-1}{n}\right)^2 a\beta_T^2 - \overline{u}.$$

Maximizing with respect to β_T yields

$$\beta_T = \frac{n}{n-1}.$$

Hence,

$$b_T = \overline{u} + \frac{1}{2}a,$$

and expected profit per agent at the optimum tournament is

$$E\Pi_T = \frac{3}{2}a - \overline{u},$$

which is identical to that under piece rates. The intuition here is that when the agents are risk-neutral the principal cannot charge them a risk-premium for removing common uncertainty from their responsibility, and the contract induces the agents to exert the same effort $e_i = a$ they would exert under piece rates. This result conforms with the Lazear and Rosen (1981) finding that both piece rates and tournaments are equally efficient with risk-neutral agents.[19]

We continue with the case of liquidity constraints for the agents. In this case, under piece rates, it follows that

$$b_R + \beta_R x_i \geq \underline{w} \Leftrightarrow \overline{u} + \frac{1}{2}a + \eta + \varepsilon_i \geq \underline{w}.$$

Thus, in the worst state of nature, the liquidity constraint must satisfy

$$\overline{u} + \frac{1}{2}a - c \geq \underline{w}.$$

If $\underline{w}$ satisfies this inequality, the optimal piece rate contract is the one determined above without liquidity constraints. By contrast, for a sufficiently large $\underline{w}$, the liquidity constraint is binding. Thus,

$$b_R = \underline{w} - \beta_R(a - c),$$

while the optimal effort is still given by (2). Expected profit per agent satisfies

$$E\Pi_R = a - a\beta_R^2 - \underline{w} + \beta_R(a - c).$$

Maximizing with respect to β_R implies

$$\beta_R = \frac{a - c}{2a}.$$

19 Again, note that Lazear and Rosen (1981) considered rank-order tournaments though.

It can be shown that the individual rationality constraint yields

$$c + \frac{1}{2}a - \frac{3}{2}\frac{c^2}{a} \geq 4\left(\overline{u} - \underline{w}\right).$$

If the individual rationality constraint is binding, β_R is calculated from the constraint with equality. That is,

$$\underline{w} - \beta_R(a-c) + \frac{1}{2}a\beta_R^2 + a\beta_R = \overline{u} \Leftrightarrow \frac{1}{2}a\beta_R^2 + c\beta_R + \left(\underline{w} - \overline{u}\right) = 0$$
$$\Leftrightarrow \beta_R = \frac{-c + \sqrt{c^2 - 2a\left(\underline{w} - \overline{u}\right)}}{a},$$

because β_R cannot be negative and provide correct incentives.

Under tournaments, the liquidity constraint implies

$$b_T + \beta_T\left(x_i - \overline{x}\right) \geq \underline{w} \Leftrightarrow b_T + \beta_T\left(e_i - \overline{e}\right) + \beta_T\left(\varepsilon_i - \overline{\varepsilon}\right) \geq \underline{w}.$$

Given agent homogeneity, it follows that

$$b_T + \beta_T\left(\varepsilon_i - \overline{\varepsilon}\right) \geq \underline{w} \Leftrightarrow \overline{u} + \frac{1}{2}a + \frac{n}{n-1}\left(\varepsilon_i - \overline{\varepsilon}\right) \geq \underline{w}.$$

Thus, in the worst state of nature, the liquidity constraint must satisfy

$$\overline{u} + \frac{1}{2}a - c \geq \underline{w}.$$

If $\underline{w}$ satisfies this inequality, the optimal tournament contract is the one determined above without liquidity constraints. Similar to the piece rate case, the liquidity constraint is binding for a sufficiently large $\underline{w}$. Thus,

$$b_T = \underline{w} + \beta_T d,$$

while the optimal effort is still given by (9). Expected profit per agent satisfies

$$E\Pi_T = \left(\frac{n-1}{n}a - d\right)\beta_T + a - \underline{w},$$

where, similar to Lemma 2 above, we assume that the regularity condition $\frac{n-1}{n}a - d > 0$ holds. If the individual rationality constraint is binding, β_T is calculated from the constraint with equality. That is,

$$\underline{w} + \beta_T d - \frac{\left(\frac{n-1}{n} a\beta_T\right)^2}{2a} = \overline{u} \Leftrightarrow \frac{1}{2}\left(\frac{n-1}{n}\right)^2 a\beta_T^2 - d\beta_T + \left(\overline{u} - \underline{w}\right) = 0$$

$$\Leftrightarrow \beta_T = \frac{d + \sqrt{d^2 - 2\left(\frac{n-1}{n}\right)^2 a\left(\overline{u} - \underline{w}\right)}}{\left(\frac{n-1}{n}\right)^2 a},$$

because the principal will obviously choose the highest-power incentives that are feasible.

To summarize the analysis with risk-neutral agents, the optimal piece rate under liquidity constraints for the agents is determined according to the following procedure:

(i) Set $\beta_R = 1$ and calculate $b_R = \overline{u} - \frac{3}{2}a$.
(ii) If $b_R + \beta_R(a - c) \geq \underline{w}$, then the liquidity constraint is always non-binding (while the individual rationality constraint is binding) and the optimal piece rate contract is the one determined above without a liquidity constraint. Otherwise, we continue with step (iii).
(iii) Calculate $\beta_R = \frac{a-c}{2a}$ and $b_R = \underline{w} - \beta_R(a - c)$.
(iv) If $c\beta_R + \frac{1}{2}a\beta_R \geq \overline{u} - \underline{w}$, then the individual rationality constraint is non binding (while the liquidity constraint is binding) and the optimal piece rate contract is the one calculated in step (iii). Otherwise, we continue with step (v).
(v) Calculate $\beta_R = \frac{-c+\sqrt{c^2-2a(\underline{w}-\overline{u})}}{a}$ and $b_R = \underline{w} - \beta_R(a - c)$, which define the optimal piece rate contract in this case (where all constraints are binding). The expected profit per agent can then be calculated as

$$E\Pi_R = a - a(\beta_R)^2 - b_R.$$

The related procedure under tournaments is:

(i) Set $\beta_T = \frac{n}{n-1}$ and calculate $b_T = \overline{u} + \frac{1}{2}a$.
(ii) If $b_T - d\beta_T \geq \underline{w}$, then the liquidity constraint is always non-binding (while the individual rationality constraint is binding) and the optimal tournament contract is the one determined above without a liquidity constraint. Otherwise, we continue with step (iii).
(iii) Calculate $\beta_T = \frac{d+\sqrt{d^2-2\left(\frac{n-1}{n}\right)^2 a(\overline{u}-\underline{w})}}{\left(\frac{n-1}{n}\right)^2 a}$ and $b_T = \underline{w} + \beta_T d$, which define the optimal tournament contract in this case (where all constraints are binding). The expected profit per agent can then be calculated as

$$E\Pi_T = \frac{n-1}{n}a\beta_T + a - b_T.$$

Figure 2 summarizes the findings of the risk-neutral agent case. As Fig. 2 shows, expected profit is identical for piece rates and tournaments when the liquidity constraints are non-binding, however, when the liquidity constraints are binding, expected

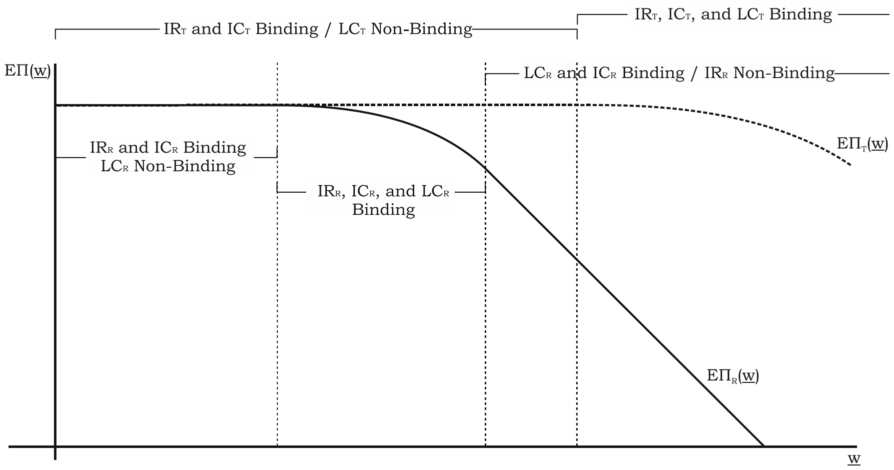

Fig. 2 The expected profit per agent for the piece rate contract and the tournament, when agents are risk-neutral

profit under tournaments is higher. These findings enhance the generality and robustness of our main result.

7 Conclusion

A familiar result in the principal-agent literature is that when agents are risk averse and production is subject to relatively large common shocks the tournament is a superior compensation scheme to the piece rate. The superiority of tournaments over piece rates may not survive under liquidity constraints. Prior research (for instance, Tsoulouhas and Vukina 1999; Marinakis and Tsoulouhas 2009 for limited liability on the principal) would lead someone to expect the same result even when limited liability is imposed on the agent instead of the principal. In addition, one might also expect that limited liability would make the agents more tolerant to risk (in the sense that liquidity constraints convexify the agent's utility function) and the principal less tolerant to risk (in the sense that the principal cares about the allocation of payments across states in order to satisfy the liquidity constraints). The reduced interest of agents in getting insurance, as well as the reduced ability of the principal to provide it, might diminish the scope for tournaments. However, there is a fundamental difference between limited liability on the principal side and limited liability on the agent side. Under limited liability for the principal, the agents cannot be suckered by the prospect of payments the principal cannot make, therefore, a limited liability constraint is introduced to put a maximum on the payment to the agents in low states. Liquidity constraints for the agents, instead, put a minimum on the payments to the agents in low states. Our analysis builds on this fact to show that in the presence of common uncertainty a principal contracting with risk averse agents will prefer to offer a tournament even when agents are liquidity constrained.

The rationale for our result is that by providing insurance against common shocks through a cardinal tournament, so that payments to the agents in unfavorable states increase and payments in favorable states decrease, the principal can satisfy tight liquidity constraints for the agents without paying any ex ante rents to them while simultaneously providing them with higher-power incentives than under piece rates. The larger the minimum payment satisfying an agent's liquidity constraint, the higher the power of incentives the principal provides. In other words, the principal counterbalances the increase in the base payment, which is required to satisfy the liquidity constraint, with higher-power incentives in order to curb agent rents and in order to reduce the likelihood that output is low. Tournaments provide the principal with added flexibility in the determination of this power.

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Appendix A

This following table (Table 1) presents the main results of a representative numerical analysis when common uncertainty is sufficiently large that is, when $[n/(n-1)]d < c$. In particular, for the parameter set $a = 10$, $n = 100$, $c = 3$, $d = 0.5$, we derive the vector of contractual parameters for a specified range of $\underline{w}$.

Appendix B

This following table (Table 2) presents the results of a numerical analysis if common uncertainty were relatively low, that is, if $[n/(n-1)]d > c$. In particular, for the parameter set $a = 10$, $n = 100$, $c = 1$, $d = 3$, we derive the vector of contractual parameters for a specified range of $\underline{w}$.

Table 1 Parameter values: $a = 10$, $n = 100$, $c = 3$, $d = 0.5$

Minimum wage	bR	βR	ΠR	IR for PR	LC for PR	bT	βT	ΠT	IR for Tour.	LC for Tour.
1	−12.4362	0.8929	14.4643	Binding	Non-binding	4.9019	0.9814	14.9019	Binding	Non-binding
1.1	−12.4362	0.8929	14.4643	Binding	Non-binding	4.9019	0.9814	14.9019	Binding	Non-binding
1.2	−12.4362	0.8929	14.4643	Binding	Non-binding	4.9019	0.9814	14.9019	Binding	Non-binding
1.3	−12.4362	0.8929	14.4643	Binding	Non-binding	4.9019	0.9814	14.9019	Binding	Non-binding
1.4	−12.4362	0.8929	14.4643	Binding	Non-binding	4.9019	0.9814	14.9019	Binding	Non-binding
1.5	−12.4362	0.8929	14.4643	Binding	Non-binding	4.9019	0.9814	14.9019	Binding	Non-binding
1.6	−12.4362	0.8929	14.4643	Binding	Non-binding	4.9019	0.9814	14.9019	Binding	Non-binding
1.7	−12.4362	0.8929	14.4643	Binding	Non-binding	4.9019	0.9814	14.9019	Binding	Non-binding
1.8	−4.3776	0.8825	6.5893	Binding	Binding	4.9019	0.9814	14.9019	Binding	Non-binding
1.9	−4.3763	0.8966	6.3371	Binding	Binding	4.9019	0.9814	14.9019	Binding	Non-binding
2	−4.3729	0.9104	6.0844	Binding	Binding	4.9019	0.9814	14.9019	Binding	Non-binding
2.1	−4.3675	0.9239	5.8311	Binding	Binding	4.9019	0.9814	14.9019	Binding	Non-binding
2.2	−4.3602	0.9372	5.5773	Binding	Binding	4.9019	0.9814	14.9019	Binding	Non-binding
2.3	−4.3512	0.9502	5.323	Binding	Binding	4.9019	0.9814	14.9019	Binding	Non-binding
2.4	−4.3405	0.9629	5.0682	Binding	Binding	4.9019	0.9814	14.9019	Binding	Non-binding
2.5	−4.3283	0.9755	4.8129	Binding	Binding	4.9019	0.9814	14.9019	Binding	Non-binding
2.6	−4.3146	0.9878	4.5571	Binding	Binding	4.9019	0.9814	14.9019	Binding	Non-binding
2.7	−4.2995	0.9999	4.301	Binding	Binding	4.9019	0.9814	14.9019	Binding	Non-binding
2.8	–	–	–	PR not defined		4.9019	0.9814	14.9019	Binding	Non-binding
2.9	–	–	–	PR not defined		4.9019	0.9814	14.9019	Binding	Non-binding
3	–	–	–	PR not defined		4.9019	0.9814	14.9019	Binding	Non-binding
3.1	–	–	–	PR not defined		4.9019	0.9814	14.9019	Binding	Non-binding
3.2	–	–	–	PR not defined		4.9019	0.9814	14.9019	Binding	Non-binding
3.3	–	–	–	PR not defined		4.9019	0.9814	14.9019	Binding	Non-binding
3.4	–	–	–	PR not defined		4.9019	0.9814	14.9019	Binding	Non-binding
3.5	–	–	–	PR not defined		4.9019	0.9814	14.9019	Binding	Non-binding
3.6	–	–	–	PR not defined		4.9019	0.9814	14.9019	Binding	Non-binding
3.7	–	–	–	PR not defined		4.9019	0.9814	14.9019	Binding	Non-binding
3.8	–	–	–	PR not defined		4.9019	0.9814	14.9019	Binding	Non-binding
3.9	–	–	–	PR not defined		4.9019	0.9814	14.9019	Binding	Non-binding
4	–	–	–	PR not defined		4.9019	0.9814	14.9019	Binding	Non-binding

Table 1 continued

Minimum wage	bR	βR	ΠR	IR for PR	LC for PR	bT	βT	ΠT	IR for Tour.	LC for Tour.
4.1	–	–	–	PR not defined		4.9019	0.9814	14.9019	Binding	Non-binding
4.2	–	–	–	PR not defined		4.9019	0.9814	14.9019	Binding	Non-binding
4.3	–	–	–	PR not defined		4.9019	0.9814	14.9019	Binding	Non-binding
4.4	–	–	–	PR not defined		4.9019	0.9814	14.9019	Binding	Non-binding
4.5	–	–	–	PR not defined		4.9984	0.9967	14.9589	Binding	Binding
4.6	–	–	–	PR not defined		5.1036	1.0072	14.9579	Binding	Binding
4.7	–	–	–	PR not defined		5.2087	1.0175	14.9559	Binding	Binding
4.8	–	–	–	PR not defined		5.3138	1.0277	14.9528	Binding	Binding
4.9	–	–	–	PR not defined		5.4189	1.0378	14.9488	Binding	Binding
5	–	–	–	PR not defined		5.5239	1.0478	14.9437	Binding	Binding
5.1	–	–	–	PR not defined		5.6289	1.0577	14.9378	Binding	Binding
5.2	–	–	–	PR not defined		5.7338	1.0675	14.9309	Binding	Binding
5.3	–	–	–	PR not defined		5.8386	1.0772	14.9231	Binding	Binding
5.4	–	–	–	PR not defined		5.9434	1.0869	14.9145	Binding	Binding
5.5	–	–	–	PR not defined		6.0482	1.0964	14.905	Binding	Binding
5.6	–	–	–	PR not defined		6.1529	1.1059	14.8947	Binding	Binding
5.7	–	–	–	PR not defined		6.2576	1.1152	14.8836	Binding	Binding
5.8	–	–	–	PR not defined		6.3623	1.1245	14.8717	Binding	Binding
5.9	–	–	–	PR not defined		6.4669	1.1337	14.8591	Binding	Binding
6	–	–	–	PR not defined		6.5714	1.1429	14.8457	Binding	Binding
6.1	–	–	–	PR not defined		6.676	1.1519	14.8317	Binding	Binding
6.2	–	–	–	PR not defined		6.7804	1.1609	14.8169	Binding	Binding
6.3	–	–	–	PR not defined		6.8849	1.1698	14.8014	Binding	Binding
6.4	–	–	–	PR not defined		6.9893	1.1786	14.7853	Binding	Binding
6.5	–	–	–	PR not defined		7.0937	1.1874	14.7685	Binding	Binding
6.6	–	–	–	PR not defined		7.1981	1.1961	14.7511	Binding	Binding
6.7	–	–	–	PR not defined		7.3024	1.2048	14.7331	Binding	Binding
6.8	–	–	–	PR not defined		7.4067	1.2133	14.7144	Binding	Binding
6.9	–	–	–	PR not defined		7.5109	1.2218	14.6952	Binding	Binding
7	–	–	–	PR not defined		7.6151	1.2303	14.6754	Binding	Binding

Table 2 Parameter values: $a = 10$, $n = 100$, $c = 1$, $d = 3$

Minimum wage	bR	βR	ΠR	IR for PR	LC for PR	bT	βT	ΠT	IR for Tour.	LC for Tour.
1	−14.0533	0.9615	14.8077	Binding	Non-binding	4.4479	0.9202	14.4479	Binding	Non-binding
1.1	−14.0533	0.9615	14.8077	Binding	Non-binding	4.4479	0.9202	14.4479	Binding	Non-binding
1.2	−14.0533	0.9615	14.8077	Binding	Non-binding	4.4479	0.9202	14.4479	Binding	Non-binding
1.3	−14.0533	0.9615	14.8077	Binding	Non-binding	4.4479	0.9202	14.4479	Binding	Non-binding
1.4	−14.0533	0.9615	14.8077	Binding	Non-binding	4.4479	0.9202	14.4479	Binding	Non-binding
1.5	−14.0533	0.9615	14.8077	Binding	Non-binding	4.4479	0.9202	14.4479	Binding	Non-binding
1.6	−14.0533	0.9615	14.8077	Binding	Non-binding	4.4479	0.9202	14.4479	Binding	Non-binding
1.7	−14.0533	0.9615	14.8077	Binding	Non-binding	4.2321	0.844	13.927	Binding	Binding
1.8	−14.0533	0.9615	14.8077	Binding	Non-binding	4.3761	0.8587	13.9248	Binding	Binding
1.9	−14.0533	0.9615	14.8077	Binding	Non-binding	4.5192	0.8731	13.9204	Binding	Binding
2	−14.0533	0.9615	14.8077	Binding	Non-binding	4.6613	0.8871	13.9141	Binding	Binding
2.1	−14.0533	0.9615	14.8077	Binding	Non-binding	4.8026	0.9009	13.9058	Binding	Binding
2.2	−14.0533	0.9615	14.8077	Binding	Non-binding	4.9431	0.9144	13.8959	Binding	Binding
2.3	−14.0533	0.9615	14.8077	Binding	Non-binding	5.0829	0.9276	13.8842	Binding	Binding
2.4	−14.0533	0.9615	14.8077	Binding	Non-binding	5.222	0.9407	13.871	Binding	Binding
2.5	−14.0533	0.9615	14.8077	Binding	Non-binding	5.3604	0.9535	13.8563	Binding	Binding
2.6	−14.0533	0.9615	14.8077	Binding	Non-binding	5.4981	0.966	13.8402	Binding	Binding
2.7	−14.0533	0.9615	14.8077	Binding	Non-binding	5.6353	0.9784	13.8228	Binding	Binding
2.8	−14.0533	0.9615	14.8077	Binding	Non-binding	5.7719	0.9906	13.8041	Binding	Binding
2.9	−14.0533	0.9615	14.8077	Binding	Non-binding	5.9079	1.0026	13.7842	Binding	Binding
3	−14.0533	0.9615	14.8077	Binding	Non-binding	6.0434	1.0145	13.7631	Binding	Binding
3.1	−14.0533	0.9615	14.8077	Binding	Non-binding	6.1784	1.0261	13.7409	Binding	Binding
3.2	−14.0533	0.9615	14.8077	Binding	Non-binding	6.313	1.0377	13.7177	Binding	Binding

Table 2 continued

Minimum wage	bR	βR	ΠR	IR for PR	LC for PR	bT	βT	ΠT	IR for Tour.	LC for Tour.
3.3	−14.0533	0.9615	14.8077	Binding	Non-binding	6.447	1.049	13.6934	Binding	Binding
3.4	−14.0533	0.9615	14.8077	Binding	Non-binding	6.5807	1.0602	13.6682	Binding	Binding
3.5	−14.0533	0.9615	14.8077	Binding	Non-binding	6.7139	1.0713	13.642	Binding	Binding
3.6	−14.0533	0.9615	14.8077	Binding	Non-binding	6.8467	1.0822	13.6149	Binding	Binding
3.7	−14.0533	0.9615	14.8077	Binding	Non-binding	6.9791	1.093	13.587	Binding	Binding
3.8	−14.0533	0.9615	14.8077	Binding	Non-binding	7.1112	1.1037	13.5582	Binding	Binding
3.9	−4.9186	0.9798	5.3177	Binding	Binding	7.2429	1.1143	13.5286	Binding	Binding
4	−4.9184	0.9909	5.0989	Binding	Binding	7.3742	1.1247	13.4982	Binding	Binding
4.1	–	–	–	PR not defined		7.5052	1.1351	13.4671	Binding	Binding
4.2	–	–	–	PR not defined		7.6359	1.1453	13.4352	Binding	Binding
4.3	–	–	–	PR not defined		7.7662	1.1554	13.4027	Binding	Binding
4.4	–	–	–	PR not defined		7.8962	1.1654	13.3694	Binding	Binding
4.5	–	–	–	PR not defined		8.026	1.1753	13.3355	Binding	Binding
4.6	–	–	–	PR not defined		8.1554	1.1851	13.301	Binding	Binding
4.7	–	–	–	PR not defined		8.2846	1.1949	13.2658	Binding	Binding
4.8	–	–	–	PR not defined		8.4135	1.2045	13.23	Binding	Binding
4.9	–	–	–	PR not defined		8.5421	1.214	13.1936	Binding	Binding
5	–	–	–	PR not defined		8.6705	1.2235	13.1567	Binding	Binding
5.1	–	–	–	PR not defined		8.7986	1.2329	13.1192	Binding	Binding
5.2	–	–	–	PR not defined		8.9265	1.2422	13.0811	Binding	Binding
5.3	–	–	–	PR not defined		9.0542	1.2514	13.0426	Binding	Binding
5.4	–	–	–	PR not defined		9.1816	1.2605	13.0035	Binding	Binding
5.5	–	–	–	PR not defined		9.3088	1.2696	12.9639	Binding	Binding

Table 2 continued

Minimum wage	bR	βR	ΠR	IR for PR	LC for PR	bT	βT	ΠT	IR for Tour.	LC for Tour.
5.6	–	–	–	PR not defined		9.4357	1.2786	12.9239	Binding	Binding
5.7	–	–	–	PR not defined		9.5625	1.2875	12.8833	Binding	Binding
5.8	–	–	–	PR not defined		9.689	1.2963	12.8423	Binding	Binding
5.9	–	–	–	PR not defined		9.8154	1.3051	12.8008	Binding	Binding
6	–	–	–	PR not defined		9.9415	1.3138	12.7589	Binding	Binding
6.1	–	–	–	PR not defined		10.0675	1.3225	12.7166	Binding	Binding
6.2	–	–	–	PR not defined		10.1932	1.3311	12.6738	Binding	Binding
6.3	–	–	–	PR not defined		10.3188	1.3396	12.6307	Binding	Binding
6.4	–	–	–	PR not defined		10.4442	1.3481	12.5871	Binding	Binding
6.5	–	–	–	PR not defined		10.5694	1.3565	12.5431	Binding	Binding
6.6	–	–	–	PR not defined		10.6944	1.3648	12.4988	Binding	Binding
6.7	–	–	–	PR not defined		10.8193	1.3731	12.454	Binding	Binding
6.8	–	–	–	PR not defined		10.944	1.3813	12.4089	Binding	Binding
6.9	–	–	–	PR not defined		11.0686	1.3895	12.3635	Binding	Binding
7	–	–	–	PR not defined		11.1929	1.3976	12.3176	Binding	Binding

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