E C O N O M I C S   B U L L E T I N

Tournaments with Ex Post Heterogeneous Agents

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Abstract

This paper compares relative performance evaluation via tournaments to absolute performance evaluation via piece rates when agents are heterogeneous ex post, to make the point that agent heterogeneity compromises the insurance function of tournaments. In particular, we show that the more heterogeneous agents are the less insurance can be offered through tournaments and the less dominant tournaments are over piece rates. Thus, absolute performance piece rates should be preferred when agents are highly heterogeneous. However, even with heterogeneous agents, tournaments become more desirable when the number of agents or the uncertainty about the common shock increases sufficiently.

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1. Introduction

Beginning with the seminal work of Lazear and Rosen (1981), Holmström (1982), Green and Stockey (1983) and Nalebuff and Stiglitz (1983), contrasting tournaments to piece rates mirrors contrasting relative performance to absolute performance evaluation. Relative performance evaluation is justified by the fact that, when agent production activities are subject to a common shock, individual performance is not a *sufficient statistic* for individual effort. The performance levels obtained by the rest of the agents convey an informative signal about the common shock and, therefore, the effort choice of any given agent. Lazear and Rosen, in particular, show that when agents have a CARA utility function and there is no common uncertainty piece rates dominate tournaments. By contrast, with large common uncertainty tournaments dominate. The rationale for this highly acclaimed result is that tournaments allow the principal to remove the common shock from the responsibility of agents. Tournaments constitute a move closer to the First Best because the principal uses the available information more efficiently. By removing common uncertainty from the responsibility of agents, and by charging a premium for this insurance, the principal increases his profit without hurting the agents. What has received little attention, though, is that when agents are heterogeneous relative performance evaluation via tournaments exposes agents to uncertainty about the average agent ability.[1]

We measure agent heterogeneity by the variance of agent ability. When this variance increases, agents are more heterogeneous because the realizations of their ability types are drawn from a more disperse distribution and, at the same time, the variability in individual output increases. The analysis then shows that the more heterogeneous agents are the less insurance can be offered through tournaments and the less dominant tournaments are over piece rates. Thus, tournaments become less desirable when the variance of the distribution of ability types is large, so that absolute performance piece rates are preferred when agents are highly heterogeneous. However, even with heterogeneous agents, tournaments become more desirable when the number of agents or the uncertainty about the common shock increases sufficiently. The rationale is that, unlike piece rates, tournaments filter away common uncertainty from the responsibility of agents who pay a premium for this insurance. Tournaments expose agents to the idiosyncratic shocks of other agents, but the average idiosyncratic shock is nullified with more agents because some shocks will be positive and some will be negative.

[1] Konrad and Kovenock (2006) examine discriminating contests in which contestants' abilities are stochastic but become common knowledge before agents choose efforts. Tsoulouhas *et al* (2007) consider CEO contests that are open to heterogeneous outsider contestants, to analyze the trade off between incentives and selection. Kolmar and Sisak (2007) analyze discriminating contests among heterogeneous contestants to guarantee efficient contributions to a public good. Riis (2007) shows that when agents are heterogeneous ex ante and the optimal discriminatory prize premium is non-monotonic in ability, efficiency can be restored if agents choose from a menu.

Both piece rates and tournaments expose an agent to uncertainty about his own ability but only tournaments expose him to uncertainty from the fact that his ability may differ from the average ability of other agents. Thus, the removal of common uncertainty from the agents' responsibility under tournament has a negative impact on their required compensation but, on the other hand, the introduction of uncertainty due to heterogeneity has a positive impact. When the variance of ability is relatively large, the addition of uncertainty from heterogeneity can outweigh the reduction of uncertainty from common shocks. Further, in this case common uncertainty against which tournaments can insure is a smaller fraction of total uncertainty.

2. The model

A principal signs a contract with n agents. Agent i produces output according to the production function $x_i = a_i + e_i + \eta + \varepsilon_i$, where a_i is the agent's ability, e_i is his effort, η is a common shock inflicted on all agents and ε_i is an idiosyncratic shock. Agents do not know their ability types at the time of contracting; instead, they privately learn their types ex post. In particular, agents discover how good they are in this activity after contracts are stipulated and before selecting effort.[2] Each agent's effort and the subsequent realizations of the production shocks are private information, but the output obtained is publicly observed. The price of output is normalized to 1. Each agent's type follows an i.i.d. normal distribution with mean μ and variance σ_a^2. The common shock follows a normal distribution with mean zero and variance σ_η^2. The idiosyncratic shock for each agent follows an i.i.d. normal distribution with mean zero and variance σ_ε^2. The distributions of a_i, η and ε_i are independent. The principal compensates agents for their effort based on their outputs by using a piece rate contract or a tournament. Agent preferences are represented by a CARA utility function $u(w_i, e_i) = -\exp\left(-rw_i + \frac{r}{2}e_i^2\right)$, where r is the agent's coefficient of absolute risk aversion and w_i is the compensation to agent i. Given the normality assumptions above, both x_i and w_i are normally distributed and, therefore, $u(\cdot)$ follows a lognormal distribution, which allows us to obtain analytical solutions.

3. The piece rate contract

The piece rate contract (R) is the payment scheme in which the compensation to agent i takes the form $w_i = b_R + \beta_R x_i$. The principal determines the contractual parameters by

[2] This is more interesting than the case when individual abilities are observed after efforts are exerted, because ability could then be summed up with all idiosyncratic shocks. Further, abstracting from the case when agents know their types at the time of contracting allows us to examine the full impact of heterogeneity on the form of the contract. Under adverse selection, instead, if the principal offered a menu of contracts and agents self-selected, agents might not be exposed to the full impact of uncertainty about average ability. See Bhattacharya and Guash (1988) and Riis (2007) for such heterogeneity.

backward induction. First, the principal calculates each agent's expected utility after he signs the contract and learns his type but before exerting effort:

$$EU_R(e_i|a_i, b_R, \beta_R) = -E\left[\exp\left(-rw_i + \frac{r}{2}e_i^2\right)\right] =$$

$$= -\exp\left[-r\left(b_R + \beta_R(a_i + e_i) - \frac{1}{2}e_i^2\right)\right] E\left[\exp\left[-r\beta_R\left(\varepsilon_i + \eta\right)\right]\right] =$$

$$= -\exp\left[-r\left[b_R + \beta_R(a_i + e_i) - \frac{e_i^2}{2} - \frac{r\beta_R^2(\sigma_\eta^2 + \sigma_\varepsilon^2)}{2}\right]\right], \tag{1}$$

where the expression in the square brackets is the certainty equivalent compensation. To ensure the compatibility of the contract with agent incentives to perform, the principal calculates the effort level that maximizes (1). First order conditions yield

$$e_i^* = \beta_R, \tag{2}$$

which implies that incentives to perform are fully determined by the incentives provided by the principal via the piece rate β_R. Note that, when both ability and effort add to output and hence to expected compensation, ability does not affect the effort choice of the agent. Given (2), to ensure the compatibility of the contract with agent incentives to participate before they learn their types, the principal selects the value of the base payment, b_R, that satisfies the agent's individual rationality constraint with equality so that he receives no rents but still accepts the contract. For ease of exposition, we normalize his reservation utility to -1, hence:

$$EU_R(b_R, \beta_R|e_i^*) = -\exp\left[-r\left[b_R + \beta_R(\mu + e_i^*) - \frac{e_i^{*2}}{2} - \frac{r\beta_R^2(\sigma_a^2 + \sigma_\eta^2 + \sigma_\varepsilon^2)}{2}\right]\right] = -1 \iff$$

$$\iff b_R = \frac{r(\sigma_a^2 + \sigma_\eta^2 + \sigma_\varepsilon^2) - 1}{2}\beta_R^2 - \mu\beta_R. \tag{3}$$

By setting b_R in accordance with (3) the principal can induce agent participation at least cost. Given conditions (2) and (3) the principal maximizes expected total profit

$$ET\Pi_R = \sum_{i=1}^{n}\left[Ex_i - Ew_i\right] = n\left[\mu + \beta_R - \frac{r(\sigma_a^2 + \sigma_\eta^2 + \sigma_\varepsilon^2) + 1}{2}\beta_R^2\right]. \tag{4}$$

The solution satisfies

$$\beta_R = \frac{1}{1 + r(\sigma_a^2 + \sigma_\eta^2 + \sigma_\varepsilon^2)}, \tag{5}$$

that is, the larger the variances of ability, common shock and idiosyncratic shock, the lower the piece rate because the weaker the link between the power of incentives and output. Condition (3) then implies

$$b_R = -\frac{r(\sigma_a^2 + \sigma_\varepsilon^2 + \sigma_\eta^2)\,(2\mu - 1) + 2\mu + 1}{2\left(1 + r(\sigma_a^2 + \sigma_\varepsilon^2 + \sigma_\eta^2)\right)^2}, \tag{6}$$

that is, expected agent ability has a negative impact on the base payment because more able agents need weaker incentives to participate. The principal's expected profit per agent under the piece rate is

$$E\Pi_R = \mu + \frac{1}{2(1 + r(\sigma_a^2 + \sigma_\varepsilon^2 + \sigma_\eta^2)}. \tag{7}$$

4. The tournament

The tournament (T) is the payment scheme in which the compensation to each agent is determined by relative performance. Specifically,

$$w_i = b_T + \beta_T(x_i - \overline{x}) = b_T + \beta_T\left(\frac{n-1}{n}x_i - \frac{1}{n}\sum_{j\neq i} x_j\right), \tag{8}$$

where $\overline{x}$ is the average output obtained by all agents. Note that the total wage bill is proportional to the base payment b_T: $\Sigma w_i = nb_T$. Thus, in contrast to the piece rate contract, the principal's total payment to the agents and, hence, the expected payment per agent are independent of output, however, each agent's relative performance determines his share of the fixed total payments. The agent's expected utility after he signs the contract and learns his type, but before exerting effort, is

$$EU_T(e_i|a_i, b_T, \beta_T) =$$

$$= -\exp\left[-r\left(b_T + \beta_T\left(a_i - \frac{1}{n}\sum_i a_i + e_i - \frac{1}{n}\sum_i e_i\right) - \frac{1}{2}e_i^2\right)\right]\cdot$$

$$\cdot E\left[\exp\left(-r\beta_T\left(\varepsilon_i - \frac{1}{n}\sum_i \varepsilon_i\right)\right)\right] =$$

$$= -\exp\left[-r\left[b_T + \beta_T\frac{n-1}{n}(a_i + e_i) - \beta_T\frac{1}{n}\sum_{j\neq i}(a_j + e_j) - \frac{e_i^2}{2} - \frac{n-1}{n}\frac{r\beta_T^2\sigma_\varepsilon^2}{2}\right]\right]. \tag{9}$$

The effort level that maximizes (9) satisfies

$$e_i^{**} = \frac{n-1}{n}\beta_T. \tag{10}$$

Further, the agent's individual rationality constraint before he learns his type is:

$$EU_T(b_T, \beta_T | e_i^{**}) =$$

$$= -\exp\left[-r\left(b_T + \beta_T \frac{n-1}{n} e_i^{**} - \beta_T \frac{1}{n} \sum_{j \neq i} e_j^{**} - \frac{e_i^2}{2} - \frac{n-1}{n} \frac{r\beta_T^2 \left(\sigma_a^2 + \sigma_\varepsilon^2\right)}{2}\right)\right] = -1 \iff$$

$$\iff b_T = \frac{1}{2} \frac{n-1}{n} \left(\frac{n-1}{n} + r\left(\sigma_a^2 + \sigma_\varepsilon^2\right)\right) \beta_T^2. \tag{11}$$

Then, given conditions (10) and (11), the principal maximizes expected total profit

$$ET\Pi_T = n\left[\mu + \frac{n-1}{n}\beta_T - \frac{1}{2}\frac{n-1}{n}\left(\frac{n-1}{n} + r\left(\sigma_a^2 + \sigma_\varepsilon^2\right)\right)\beta_T^2\right]. \tag{12}$$

The solution satisfies

$$\beta_T = \frac{1}{\frac{n-1}{n} + r\left(\sigma_a^2 + \sigma_\varepsilon^2\right)}. \tag{13}$$

Note that neither the bonus factor β_T nor the base payment b_T depend on σ_η^2 because the principal filters away common shocks from the responsibility of agents. Also note that

$$\beta_T > \beta_R, \tag{14}$$

that is, the removal of common uncertainty from the agent's responsibility enables the principal to implement higher-power incentives. Given (13), condition (11) implies

$$b_T = \frac{\frac{n-1}{n}}{2\left(\frac{n-1}{n} + r\left(\sigma_a^2 + \sigma_\varepsilon^2\right)\right)}. \tag{15}$$

Unlike the piece rate, the base payment does not depend on the expected agent ability under tournament. This is because the expected payment under tournament does not depend on the expected agent ability. The principal's expected profit per agent is

$$E\Pi_T = \mu + \frac{\frac{n-1}{n}}{2\left(\frac{n-1}{n} + r\left(\sigma_a^2 + \sigma_\varepsilon^2\right)\right)}. \tag{16}$$

5. The dominant contract

The principal's decision about which payment scheme to offer depends entirely on expected profits. Conditions (7) and (16) imply:

$$E\Pi_T \geq E\Pi_R \iff \frac{1}{n-1}\left(\sigma_a^2 + \sigma_\varepsilon^2\right) \leq \sigma_\eta^2. \tag{17}$$

First note that, interestingly, expected agent ability does not affect the form of the dominant contract. Second, condition (17) indicates that tournaments are optimal provided that the variance of the common shock is larger than only a fraction of the variance of agent ability and the variance of the idiosyncratic shock, where the fraction decreases when the number of agents increases. Thus, the odds would be cast in favor of tournaments if common uncertainty equaled the uncertainty in agent ability plus idiosyncratic uncertainty. By the strong law of large numbers, a large number of agents strengthens the dominance of tournaments because average agent ability converges almost surely to the population mean ability, and idiosyncratic shocks cancel out, which enables the principal to offer better insurance by filtering away common shocks from the responsibility of agents through the average output. Therefore, piece rates are dominant when the number of agents is sufficiently small, or the uncertainties in agent ability or idiosyncratic shocks are large, or when the uncertainty about the common shock is sufficiently small. Our finding invites an empirical investigation of the magnitudes of these uncertainties.

6. The dominant contract when ability affects the cost of effort

We briefly extend the analysis to the case where the cost of effort depends on ability: $u(w_i, e_i) = -\exp\left(-rw_i + \frac{r}{2a_i}e_i^2\right)$. An agent's optimal effort is now a function of ability because higher ability reduces the cost of effort. Thus,

$$e_i^* = a_i\beta_R, \tag{18}$$

for a piece rate, and

$$e_i^{**} = \frac{n-1}{n}a_i\beta_T, \tag{19}$$

for a tournament. The remaining analysis gets messy pretty quickly. In particular, under piece rates, the base payment satisfies

$$b_R = \frac{r\sigma_\varepsilon^2 + r\sigma_\eta^2 + r\left(1 + \frac{1}{2}\beta_R\right)^2\sigma_a^2 - \mu}{2}\beta_R^2 - \mu\beta_R, \tag{20}$$

where β_R solves

$$\mu - \left[r\left(\sigma_\varepsilon^2 + \sigma_\eta^2 + \sigma_a^2\right) + \mu\right]\beta_R - \frac{3}{2}r\sigma_a^2\beta_R^2 - \frac{1}{2}r\sigma_a^2\beta_R^3 = 0, \tag{21}$$

and the principal's expected profit per agent is

$$E\Pi_R = \mu - \mu\beta_R^2 - b_R. \tag{22}$$

Under tournaments, the base payment satisfies

$$b_T = \frac{1}{2}\left(\frac{n-1}{n}\right)^2 \beta_T^2 \mu + \frac{1}{2}\left(\left(\frac{n-1}{n}\beta_T\left(1-\frac{1}{2}\frac{n-1}{n}\beta_T\right)\right)^2 + \frac{n-1}{n^2}\beta_T^2\right) r\sigma_a^2 + \frac{1}{2}\frac{n-1}{n} r\sigma_\varepsilon^2 \beta_T^2, \tag{23}$$

where β_T solves

$$\frac{n-1}{n}\mu - \frac{n-1}{n}\left[\frac{n-1}{n}\mu + r\left(\sigma_a^2+\sigma_\varepsilon^2\right)\right]\beta_T + \frac{3}{2}\left(\frac{n-1}{n}\right)^3 r\sigma_a^2\beta_T^2 - \frac{1}{2}\left(\frac{n-1}{n}\right)^4 r\sigma_a^2\beta_T^3 = 0, \tag{24}$$

and the principal's expected profit per agent is

$$E\Pi_T = \mu + \frac{n-1}{n}\mu\beta_T - b_T. \tag{25}$$

The range of variance of ability over which tournaments dominate piece rates, from the perspective of the principal, is larger than the corresponding range when ability does not affect the cost of effort.[3] This can easily be demonstrated by a numerical example. We set $\sigma_\eta^2 = \sigma_\varepsilon^2 = 1$, $r = 2$, $\mu = 0.5$ and $n = 5$. As shown in Figure 1, when the cost of effort does not depend on ability, tournaments are optimal for $0 \le \sigma_a^2 < 3$, however, when ability reduces the cost of effort, tournaments are optimal for $0 \le \sigma_a^2 < 4.28$. The rationale is that, when ability affects the cost of effort, the agents are being subjected to more risk and they also behave as if they were more risk-averse (i.e., even though the coefficient of absolute risk aversion has not changed, the utility function is more concave because the cost of effort is monetized). Therefore, tournaments are optimal for higher σ_a^2 values because the principal can charge more for insurance against common shocks. Note that an increase in the variance of ability still favors piece rates, similar to the analysis above when the cost of effort did not depend on ability. Also note that the variance of ability and the variance of idiosyncratic shocks are now given different weights in determining the dominant contract, with the variance of ability being given a larger weight, unlike the case when ability did not affect effort (see condition (17) where the weights were equal). It is easy to to see this by comparing conditions (20) to (3), and (23) to (11).

7. Conclusion

The literature on tournaments or contests, including Lazear and Rosen (1981), Green and Stockey (1983) and Nalebuff and Stiglitz (1983), and more recently Konrad and Kovenock

[3] Also note that expected ability μ now affects the choice of dominant contract (but the direction depends on the parameter values). This is because when the cost of effort depends on ability, then, the choice of effort depends on ability, hence, the choice of contract depends on expected ability.

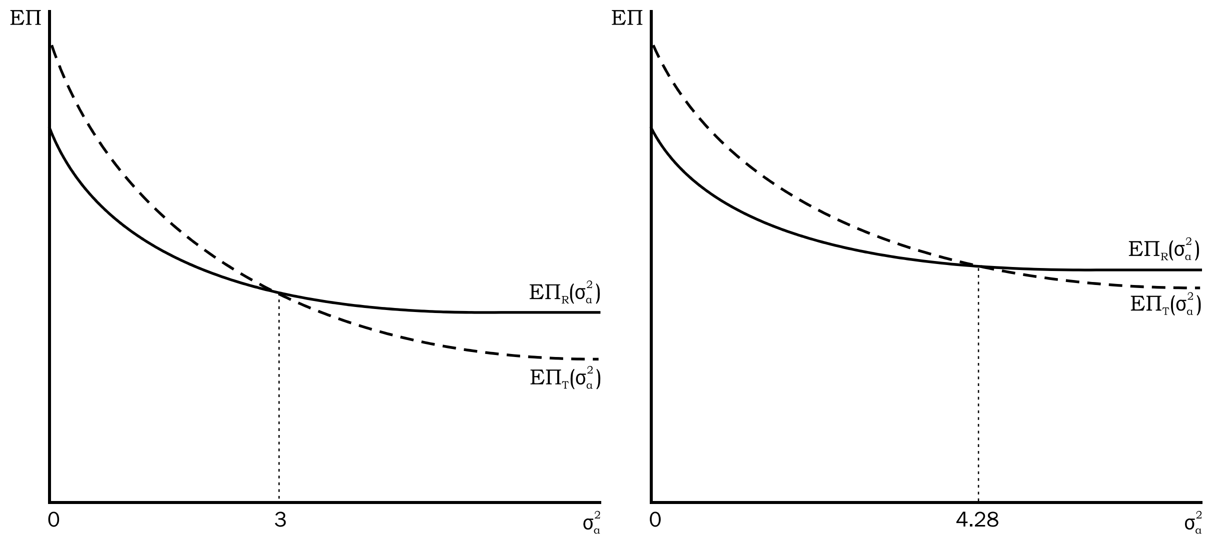

Figure 1: Expected profit per agent under piece rates and tournaments for different values of the variance of ability, when ability does not affect the cost of effort (left panel) and when it does (right panel).

(2006), Tsoulouhas *et al* (2007) and Riis (2007), has ignored the fact that when agents are heterogeneous relative performance evaluation exposes agents to uncertainty about the average agent ability. This paper compares relative performance evaluation via tournaments to absolute performance evaluation via piece rates when agents are heterogeneous ex post, and shows that agent heterogeneity compromises the insurance function of tournaments. In particular, the analysis shows that tournaments become less desirable when the variance of the distribution of ability types is large, so that absolute performance piece rates should be preferred when agents are highly heterogeneous. However, even with heterogeneous agents, tournaments become more desirable when the number of agents or the uncertainty about the common shock increases sufficiently.

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