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Practice Set 8

Economic Growth

This set contains problems for your own practice. Do not just read the provided solutions — try to solve the problems first, and use the solutions only when you are stuck. Reading solutions without attempting the problems first has the same value for your preparation as watching someone else run a marathon and then expecting to be able to run it yourself. If you are unable to comprehend the solution of a problem even after reading the instructional answer key, you have most likely missed the relevant part of the material underlying this problem. Direct any questions about this problem set to your section's teaching assistant.

1. GDP is x and it grows at 10% per year. How many years till it becomes $2x$?
2. Explain the difference between *sustained growth* and *catch-up growth*.
3. Explain the difference between *depreciation* and *depreciation rate*; and between *saving* and *saving rate*.
4. Consider an economy where the current GDP is below the steady-state GDP.
   (a) Explain the difference between *current GDP* and *steady-state GDP*.
   (b) Is this economy expected to grow or shrink next year?
   (c) Is this economy expected to grow or shrink forever?
   (d) Can the GDP of an economy be above its steady-state?
5. Explain how *sustained* and *catch-up* growth could be explained in the context of the Solow model.
6. Consider an economy with production function $Y = 5\sqrt{K}$, capital stock of \$324B, saving rate of 40%, and depreciation rate of 10%.
   (a) How much will the capital stock be next year?
   (b) Find the steady-state GDP.
7. Consider two economies with identical production functions, saving rates and depreciation rates. Explain whether the following situations are possible according to the Solow model.
   (a) The two economies have different steady-state GDPs.
   (b) The two economies currently exhibit different growth rates.

Answer Key

1. GDP is x and it grows at 10% per year. How many years till it becomes $2x$?

   If GDP grows at 10% per year:
   - The 1st year it will be $x \cdot (1 + 0.1) = 1.1x$;
   - The 2nd year it will be $x \cdot (1 + 0.1)^2 = 1.21x$;
   - The 3rd year it will be $x \cdot (1 + 0.1)^3 = 1.33x$;
   - The 4th year it will be $x \cdot (1 + 0.1)^4 = 1.46x$;
   - The 5th year it will be $x \cdot (1 + 0.1)^5 = 1.61x$;
   - The 6th year it will be $x \cdot (1 + 0.1)^6 = 1.77x$.
   - The 7th year it will be $x \cdot (1 + 0.1)^7 = 1.95x$.
   - The 8th year it will be $x \cdot (1 + 0.1)^8 = 2.14x$.
   So, it will take between 7 and 8 years.

   *There is a shortcut known as "**the rule of 70**" that allows you to calculate this faster. It states that: "the number of years it takes for GDP to double is approximately 70 divided by the growth rate". Here, 70/10=7 years — consistent with our calculation above.*

2. Explain the difference between *sustained growth* and *catch-up growth*.

   Sustained growth refers to GDP growth driven by improvements in technology and human capital productivity within a country. Catch-up growth refers to GDP growth driven by foreign investment or technological spillovers from more advanced economies.

   For example, the iPhone contributed to sustained growth in the US — it was researched and designed by an American company that captured the profits from its production. It also contributed to catch-up growth in China, where it is assembled by Chinese factories.

3. Explain the difference between *depreciation* and *depreciation rate*; and between *saving* and *saving rate*.

   Depreciation is the value of capital lost due to wear and tear or obsolescence ($d \cdot K$). The depreciation rate is the fraction of capital that depreciates per period (d). Similarly, total saving is $s \cdot Y$, while the saving rate is s.

4. Consider an economy where the current GDP is below the steady-state GDP.

   (a) Explain the difference between *current GDP* and *steady-state GDP*.

   Current GDP is what an economy currently produces. Steady-state GDP is the level of GDP the economy converges to, given its production function, saving rate, and depreciation rate.

   (b) Is this economy expected to grow or shrink next year?

   The economy will grow. When actual GDP is below steady-state GDP, new capital built ($s \cdot Y$) exceeds capital lost to depreciation ($d \cdot K$), so the capital stock increases, causing GDP to rise.

   (c) Is this economy expected to grow or shrink forever?

   It will not grow forever. $s \cdot Y$ grows at a decreasing rate due to diminishing marginal product. As the economy approaches its steady-state, the growth rate becomes progressively smaller. Once the economy reaches the steady-state, growth is zero as new investment exactly offsets depreciation.

   (d) Can the GDP of an economy be above its steady-state?

   Practically, it can, but it rarely happens. An economy cannot naturally grow past its steady-state. It can, however, find itself above it after a crisis, war, or natural disaster. For instance, suppose an economy has a GDP of 90 and is converging to a steady-state of 100. A sudden shock shifts the production function downward, dropping the steady-state to 80. Since the economy is already at 90, it is now above its new steady-state and will contract toward 80.

5. Explain how *sustained* and *catch-up* growth could be explained in the context of the Solow model.

   Catch-up growth is driven primarily by an influx of foreign direct investment. Foreign capital is added to the existing capital stock (K), pushing the economy toward higher levels of Y on the production function.

   Sustained growth, on the other hand, comes from shifting the production function upward through technological advancement or improvements in human capital, which yields higher levels of Y. Capital stock K also increases because as Y rises, investment $s \cdot Y$ rises with it.

6. Consider an economy with production function $Y = 5\sqrt{K}$, capital stock of \$324B, saving rate of 40%, and depreciation rate of 10%.

   (a) How much will the capital stock be next year?

   Annual depreciation: $0.10 \cdot \$324B = \$32.4B$.

   Output: $Y = 5\sqrt{K} = 5\sqrt{\$324B} = 5 \cdot \$18B = \$90B$.

   New capital stock: $0.4 \cdot \$90B = \$36B$.

   So, next year's capital stock: $\$324B - \$32.4B + \$36B = \$327.6B$.

   (b) Find the steady-state GDP.

   At the steady-state, $s \cdot Y = d \cdot K$, *or* $0.4Y = 0.1K$.

   Substituting the production function for Y: $0.4 \cdot 5\sqrt{K} = 0.1K$, *or* $\frac{K}{\sqrt{K}} = \frac{0.4}{0.1} \cdot 5$, *or* $\sqrt{K} = 20$, *or* $K = 20^2 = \$400B$.

   From the production function: $Y = 5\sqrt{\$400B} = 5 \cdot \$20B = \$100B$.

   Note that K is the capital ***stock*** *built up through investment over many years, while Y is the annual output produced from that capital stock.*

7. Consider two economies with identical production functions, saving rates and depreciation rates. Explain whether the following situations are possible according to the Solow model.

   (a) The two economies have different steady-state GDPs.

   Not possible. Steady-state GDP occurs where $s \cdot Y = d \cdot K$. Since the two economies share the same s, d, and production function (Y), they converge to the same steady-state K, and thus, the same steady-state GDP.

   (b) The two economies currently exhibit different growth rates.

   Possible. The two economies may have started with different levels of capital stock (K) and are therefore at different stages of convergence to the same steady-state. In this case, the economy that is closer to the steady-state will exhibit a lower growth rate.

Please report any typos, mistakes, or suggestions for improvement to kmarinakis@smu.edu.sg.