

Practice Set 10

Short-run GDP Fluctuations

This set contains problems for your own practice. Do not just read the provided solutions — try to solve the problems first, and use the solutions only when you are stuck. Reading solutions without attempting the problems first has the same value for your preparation as watching someone else run a marathon and then expecting to be able to run it yourself. If you are unable to comprehend the solution of a problem even after reading the instructional answer key, you have most likely missed the relevant part of the material underlying this problem. Direct any questions about this problem set to your section's teaching assistant.

1. Assume that $C = 100 + 0.75(Y - T)$, $I = 120$, $G = 220$, $T = 0.2Y$ and $X = M$.
 - (a) Find the equilibrium output.
 - (b) Show that if G is increased by 40 units, the equilibrium income will increase by 100 units.
 - (c) If $Y = C + I + G$ and only G increased by 40 units, where are the additional 60 units in the 100-unit change of Y coming from?
2. Explain why the intersection of the 45-degree line with the $C + I + G$ line is where the equilibrium of the commodity market lies.
3. Explain the difference between *scarce reserves* and *ample reserves* in the context of the money market.
4. Consider the commodity market and the money market for an economy. An increase in C , I or G shocks the commodity market.
 - (a) Explain what will happen to the money market if banks have scarce reserves and the Central Bank does not conduct any Open Market Operations.
 - (b) Explain whether the result you found in (a) will be contained in the money market or whether it would upset the economy further.
 - (c) Explain how the central bank can prevent a shock that was caused by C , I or G from upsetting the economy further.
5. A creditor wants to loan out \$100 at 20% interest rate. Each possible debtor has 90% probability to pay off the loan and 10% probability to default paying nothing back. The probability of default is independent among debtors.
 - (a) What is the probability for the creditor to end up losing money if she loans \$100 to a single debtor?
 - (b) What is the probability for the creditor to end up losing money if she loans the \$100 to 100 debtors at \$1 each?

Answer Key

1. Assume that $C = 100 + 0.75(Y - T)$, $I = 120$, $G = 220$, $T = 0.2Y$ and $X = M$.

(a) Find the equilibrium output.

We start from $Y = C + I + G$. Substituting C , I , and G : $Y = 100 + 0.75(Y - T) + 120 + 220$, or $Y = 440 + 0.75(Y - T)$. Substituting T : $Y = 440 + 0.75(Y - 0.2Y)$, or $Y = 440 + 0.6Y$. Solving for Y : $Y = 440/0.4 = 1,100$.

(b) Show that if G is increased by 40 units, the equilibrium income will increase by 100 units.

We start from $Y = C + I + G$. Substituting C , I , and G : $Y = 100 + 0.75(Y - T) + 120 + 260$, or $Y = 480 + 0.75(Y - T)$. Substituting T : $Y = 480 + 0.75(Y - 0.2Y)$, or $Y = 480 + 0.6Y$. Solving for Y : $Y = 480/0.4 = 1,200$. So, $\Delta Y = 100$.

(c) If $Y = C + I + G$ and only G increased by 40 units, where are the additional 60 units in the 100-unit change of Y coming from?

This paradox can be explained if we notice that C does not remain constant after the change in G .

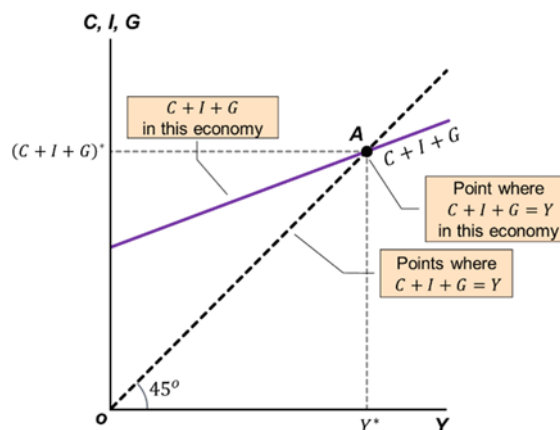
$C = 100 + 0.75(Y - T)$. Substituting T : $C = 100 + 0.75(Y - 0.2Y)$, or $C = 100 + 0.6Y$.

When $G = 220$: $C = 100 + 0.6 \cdot 1,100 = 760$, and when $G = 260$: $C = 100 + 0.6 \cdot 1,200 = 820$. So, $\Delta C = 60$, which is the additional 60 units in ΔY .

2. Explain why the intersection of the 45-degree line with the $C + I + G$ line is where the equilibrium of the commodity market lies.

The 45-degree line has one geometric property: every point on it is equidistant from both axes, meaning that for any point on the line, the value on the horizontal axis equals the value on the vertical axis.

In the commodity market graph, the horizontal axis measures production (Y) and the vertical axis measures aggregate expenditure ($C + I + G$). Equilibrium requires those two to be equal, and this is precisely the condition satisfied at every point on the 45-degree line. The $C + I + G$ line shows the expenditure at every level of Y in this economy. Only at the intersection with the 45-degree line do we have $Y = C + I + G$ — which is why it is the equilibrium of the commodity market.



3. Explain the difference between scarce reserves and ample reserves in the context of the money market.

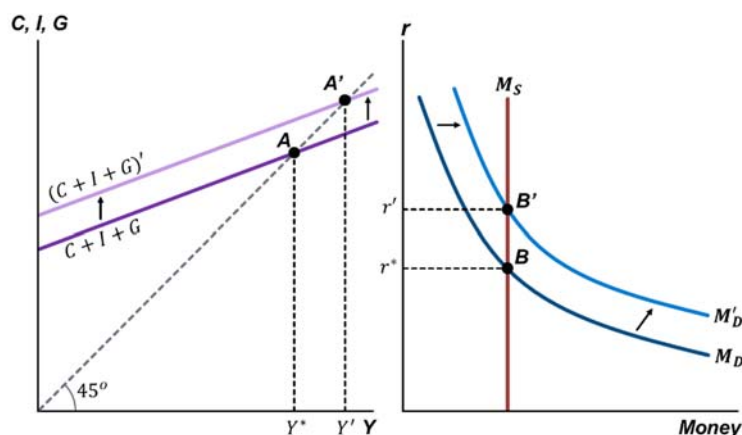
Scarce reserves mean that commercial banks cannot provide more loans to expand M_S , unless the Central Bank provides additional liquidity through Open Market Operations.

Ample reserves mean that commercial banks already have sufficient reserves to expand M_S as needed and can meet the demand for money on their own.

4. Consider the commodity market and the money market for an economy. An increase in C , I or G shocks the commodity market.

- (a) Explain what will happen to the money market if banks have scarce reserves and the Central Bank does not conduct any Open Market Operations.

The increase in C , I or G will push the $C + I + G$ line upward, increasing the equilibrium output to Y' . This will increase money demand in the money market (link 1) because individuals need to hold more money to buy and sell the additional output. Money supply will not change because banks have scarce reserves and the Central Bank does not provide additional liquidity through Open Market Purchases. The real interest rate will rise to r' .



- (b) Explain whether the result you found in (a) will be contained in the money market or whether it would upset the economy further.

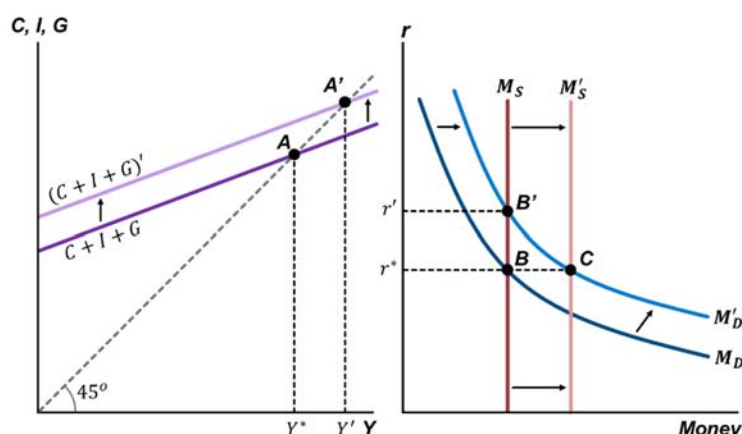
The increase in the real interest rate would cause investment to contract in the commodity market (link 2). The change in investment will again affect output and re-activate link 1. The shock will keep bouncing between the two markets, every time smaller in magnitude until it eventually dies down after creating intense short-run fluctuations.

The figure shows an example of a vicious cycle: Increase in $G \rightarrow$ Increase in $Y^* \rightarrow$ Increase in $M_D \rightarrow$ Increase in $r \rightarrow$ Decrease in $I \rightarrow$ Decrease in $Y^* \rightarrow$ Decrease in $M_D \rightarrow$ Decrease in $r \rightarrow$ Increase in $I \rightarrow$ Increase in Y^* , and the cycle keeps repeating.



- (c) Explain how the central bank can prevent a shock that was caused by C , I or G from upsetting the economy further.

The Central Bank could conduct Open Market Purchases, providing the necessary liquidity to commercial banks and enabling them to expand money supply to M'_S . This would have kept the interest rate unchanged, preventing the vicious cycle in (b) from starting. This is why Central Banks commit to controlling the real interest rate.



5. A creditor wants to loan out \$100 at 20% interest rate. Each possible debtor has 90% probability to pay off the loan and 10% probability to default paying nothing back. The probability of default is independent among debtors.

(a) What is the probability for the creditor to end up losing money if she loans \$100 to a single debtor?

The creditor will lose her \$100 if the debtor defaults (10% probability), and receive \$120 if the debtor does not default (90% probability). So, the probability of losing money is 10%.

(b) What is the probability for the creditor to end up losing money if she loans the \$100 to 100 debtors at \$1 each?

From the law of averages, out of 100 loans:

— around 10 will default, resulting in a loss of $\$1 \cdot 10 = \10 .

— around 90 will not default, resulting in a profit of $\$0.2 \cdot 90 = \18 .

The approximate net result is $\$18 - \$10 = \$8$, so the creditor is unlikely to lose money.

Of course, the law of averages does not guarantee absolute certainty. It is statistically possible that 17 or more loans will default, causing the net result to become negative. A binomial calculator finds this probability to be 0.0001%, and even if it materialized, the loss would not be the entire capital of \$100 as in (a).

Please report any typos, mistakes, or suggestions for improvement to kmarinakis@smu.edu.sg.