

Homework 4 solutions

(a)

For firm 1:

$$q_1^* = \frac{a-2c_1+c_2}{3}, P = \frac{a+c_1+c_2}{3}, \Pi_1^* = \frac{(a-2c_1+c_2)^2}{9}$$

For firm 2:

$$q_2^* = \frac{a-2c_2+c_1}{3}, P = \frac{a+c_1+c_2}{3}, \Pi_2^* = \frac{(a-2c_2+c_1)^2}{9}$$

$$\text{Total quantity } Q_{1+2} = \frac{2a-c_1-c_2}{3}$$

(b)

For firm 1:

$$q_1^* = \frac{a-3c_1+c_2+c_3}{4}, P^* = \frac{a+c_1+c_2+c_3}{4}, \Pi_1^* = \frac{(a-3c_1+c_2+c_3)^2}{16}$$

For firm 2:

$$q_2^* = \frac{a-3c_2+c_1+c_3}{4}, P^* = \frac{a+c_1+c_2+c_3}{4}, \Pi_2^* = \frac{(a-3c_2+c_1+c_3)^2}{16}$$

For firm 3:

$$q_3^* = \frac{a-3c_3+c_1+c_2}{4}, P^* = \frac{a+c_1+c_2+c_3}{4}, \Pi_3^* = \frac{(a-3c_3+c_1+c_2)^2}{16}$$

$$\text{Total quantity: } Q_{1+2+3}^* = \frac{3a-c_1-c_2-c_3}{4}$$

(c) *If cost structure is different to maximize profits under collusion the most efficient firm (assume this is firm 1) should be chosen for production (the firm that has the lowest marginal costs). It should also have a capacity to attain quantity of production optimal for the monopolist. If $c_1 = c_2$, then a good be produced by both firms.*

Assuming all firms get equal share from cartel profit:

For each firm in the two-firm cartel: $q_i^ = \frac{a-c_1}{4}$, $P = \frac{a+c_1}{2}$, $\Pi_i = \frac{(a-c_1)^2}{8}$, where $i = 1,2$*

For each firm in the three-firm cartel: $q_i^ = \frac{a-c_1}{6}$, $P = \frac{a+c_1}{2}$, $\Pi_i = \frac{(a-c_1)^2}{12}$, where $i = 1,2,3$*

In general firms have to agree on a distribution on the contract curve according to bargaining power.

(d)

For the leader:

$$q_1^* = a - c_1 - \frac{a-c_2}{2}, P = \frac{a+2c_1+c_2}{4}, \Pi_1^* = \frac{(a-2c_1+c_2)^2}{8}$$

For the follower:

$$q_2^* = \frac{3}{4}(a - c_2) - \frac{a-c_1}{2}, P = \frac{a+2c_1+c_2}{4}, \Pi_2^* = \frac{(a-3c_2+2c_1)^2}{16}$$

$$\text{Total quantity: } Q_{1+2} = \frac{3a-2c_1-c_2}{4}$$