HOMEWORK 9 SOLUTIONS

(a) Marginal consumers:

$$x_L = \frac{1}{4} + (p_M - p_L)\ (between\ firms\ L\ and\ M)$$

$$x_R = \frac{1}{4} + (p_M - p_R)\ (between\ firms\ M\ and\ R)$$

Profit functions:

$$\Pi_L = s(p_L - c)\left(\frac{1}{4} - \frac{p_M - p_L}{2t}\right)$$

$$\Pi_M = s(p_M - c)\left(\frac{1}{2} - \frac{2p_M - p_L - p_R}{2t}\right)$$

$$\Pi_R = s(p_R - c)\left(\frac{1}{4} - \frac{p_M - p_R}{2t}\right)$$

FOCs:

$$\frac{\partial \Pi_L}{\partial p_L} = S\left(\frac{1}{4} + \frac{p_M}{2t} - \frac{p_L}{t} + \frac{c}{2t}\right) = 0$$

$$\frac{\partial \Pi_M}{\partial p_M} = S\left(\frac{1}{2} - \frac{2p_M}{t} + \frac{p_L + p_R}{2t} + \frac{c}{t}\right) = 0$$

$$\frac{\partial \Pi_R}{\partial p_R} = S\left(\frac{1}{4} + \frac{p_M}{2t} - \frac{p_R}{t} + \frac{c}{2t}\right) = 0$$

$$p_L = p_M = p_R = c + \frac{t}{2}$$

$$\Pi_L^* = \Pi_R^* = \frac{st}{8}, \Pi_M^* = \frac{st}{4}$$

(b) It could enter at any point and get $\frac{1}{4}$ of the market

(c) Marginal consumers:

$$x_L = \frac{1}{8} + \frac{p_E - p_L}{2t}\ (between\ firms\ L\ and\ E)$$

$$x_E = \frac{1}{8} + \frac{p_M - p_E}{2t}\ (between\ firms\ E\ and\ M)$$

$$x_R = \frac{1}{4} + \frac{p_M - p_R}{2t}\ (between\ firms\ M\ and\ R)$$

Profit functions:

$$\Pi_L = S(p_L - c)(\frac{1}{8} + \frac{p_E - p_L}{2t})$$

$$\Pi_E = S(p_E - c)\left(\frac{1}{4} - \frac{2p_E + p_L + p_M}{2t}\right) - F$$

$$\Pi_M = S(p_M - c)(\frac{3}{8} - \frac{2p_M - p_E - p_R}{2t})$$

$$\Pi_R = S(p_R - c)(\frac{1}{4} + \frac{p_M - p_R}{2t})$$

FOCs:

$$\frac{\partial \Pi_L}{\partial p_L} = S\left(\frac{1}{8} + \frac{p_E}{2t} - \frac{p_L}{t} + \frac{c}{2t}\right) = 0$$

$$\frac{\partial \Pi_E}{\partial p_E} = S\left(\frac{1}{4} - \frac{2p_E}{t} + \frac{p_L + p_M}{2t} + \frac{c}{t}\right) = 0$$

$$\frac{\partial \Pi_M}{\partial p_M} = S\left(\frac{3}{8} - \frac{2p_M}{t} + \frac{p_E + p_R}{2t} + \frac{c}{t}\right) = 0$$

$$\frac{\partial \Pi_R}{\partial p_R} = S\left(\frac{1}{4} + \frac{p_M}{2t} - \frac{p_R}{t} + \frac{c}{2t}\right) = 0$$

$$p_L = c + \frac{4}{15}t, p_M = c + \frac{11}{30}t, p_E = c + \frac{17}{60}t, p_R = c + \frac{13}{30}t$$

$$\Pi_L = \frac{8St}{225}, \Pi_E = \frac{289St}{3600} - F, \Pi_M = \frac{121St}{900}, \Pi_R = \frac{169St}{1800}$$

(d) Assume that L and R are staying at 0 and 1 (as previously) and M and E are at $\frac{1}{3}$ and $\frac{2}{3}$

Marginal consumers:

$$x_L = \frac{1}{6} + \frac{p_M - p_L}{2t} \ (between\ firms\ L\ and\ M)$$

$$x_M = \frac{1}{6} + \frac{p_E - p_M}{2t} \ (between\ firms\ M\ and\ E)$$

$$x_R = \frac{1}{6} + \frac{p_E - p_R}{2t} \ (between\ firms\ E\ and\ R)$$

Profit functions:

$$\Pi_L = s(p_L - c)\left(\frac{1}{6} + \frac{p_M - p_L}{2t}\right)$$

$$\Pi_M = s(p_M - c)\left(\frac{1}{3} - \frac{2p_M - p_L - p_E}{2t}\right)$$

$$\Pi_E = s(p_E - c)\left(\frac{1}{3} - \frac{2p_E - p_M - p_R}{2t}\right) - F$$

$$\Pi_R = s(p_R - c)\left(\frac{1}{6} + \frac{p_E - p_R}{2t}\right)$$

Answer:

$$p_L = p_M = p_E = p_R = c + \frac{t}{3}$$

$$\Pi_L = \Pi_R = \frac{St}{18}, \Pi_M = \frac{St}{9}, \Pi_E = \frac{St}{9} - F$$

(e) If firms enter simultaneously $\Pi_E = \frac{St}{9}$, if it arrives late $\Pi_E = \frac{289St}{3600}$, so the disadvantage is $\frac{111St}{3600}$. As firm E should pay an entry fee of F is sufficient to set $F = \frac{289St}{3600}$ if E arrives late or $F = \frac{St}{9}$ of firms arrive simultaneously.

(f) Hotelling model is appropriate. Here locations could mean place on the preference scale (brand). So, this model could show what are optimal conditions for deterrence by proliferation.