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Lecture 6

Static games

Industrial Economics

Games

* ***Game*** is any situation in which the participants make ***strategic*** decisions
* Each participant of the game (***player***) has well defined objectives that ***drive*** their decisions
* For ***example***
  * Firms competing with each other by setting ***prices***
  * Individuals bidding against each other in an ***auction***

Strategic decisions

* Decisions result in ***payoffs*** to the players

  outcomes that generate ***rewards*** or ***penalties***
* In a strategic interaction, a player's ***payoff depends*** on
  1. The player's actions
  2. The opponents' actions
* To maximize payoff, every player must take their opponents' actions ***into account*** when they make their own decision
* Thus, it would be very useful for the player to understand what is the ***optimal response*** of their opponents

Noncooperative vs. cooperative games

* In ***cooperative games*** players negotiate binding contracts that allow them to plan ***joint strategies***

  example: a joint venture by two firms (i.e., HSE and NES)
* In ***non-cooperative games*** agreements are ***not binding*** or ***not allowed***

  example: Two competing firms, assuming the other's behavior, independently determine pricing strategy to gain market share

Information structure of games

* Games of ***complete information***
  * ***Everyone knows*** the structure of the game (opponents, rules, set of actions, payoffs)
  * You may ***ignore*** some past actions by rivals
* Games of ***perfect information***
  * ***Everyone knows*** the full history of actions by rivals
  * You may ***ignore*** the rules or the full set of possible payoffs
* Examples

  poker vs. competing firms' objectives

Static games of complete info

Assumptions:

* Players choose actions ***simultaneously***
* Players receive ***payoffs*** that depend on the ***combination*** of actions selected
* The payoff distribution with respect to combinations of actions is ***common knowledge***
* Players care to ***maximize*** payoffs

  all ***incentives*** and ***risk attitude*** is incorporated in those payoffs

Game bi-matrix

* To completely ***define*** a game we need to ***know 3 things***:
  * The players
  * The feasible actions for each player
  * The payoffs for each combination of actions
* All these are presented in a matrix called ***game bi-matrix***

Firm B / Firm A

Firm A	Advertise	Don't Advertise
Advertise	10, 5	15, 0
Don't Advertise	6, 8	10, 2

Strictly dominated strategies

* A strategy is ***strictly dominated*** when it yields a strictly lower payoff than another strategy, independently of the actions of the other players

P2 / P1

P1	Left	Right
Up	9 , 10	10 , 2
~~Down~~	~~2 , 3~~	~~3 , 8~~

* Rational players ***do not play*** strictly dominated strategies
* Players do not take opponents' dominated strategies ***into account***

Iterated elimination

P2 / P1

P1	Left	Middle	Right
Up	~~1 , 0~~	2 , 2	0 , 1
~~Down~~	~~0 , 3~~	~~0 , 1~~	2 , 0

* Strategy 'Right' is strictly dominated by 'Middle'
* Then, strategy 'Down' is strictly dominated by 'Up'
* Finally, dominant strategy equilibrium (DE) is (Up, Middle)

DE and IDE

* Equilibrium in dominant strategies (DE), is the outcome of the game when each player is doing ***the best it can regardless*** of what competitors are doing
* Iterative Dominant Equilibrium (IDE) results from the ***same principle*** using iterative elimination of non-best actions
* Dominant equilibriums are ***non-strategic*** concepts

  optimal strategy is determined ***without worrying*** about the actions of other players
* Notice that elimination of ***weakly dominated strategies*** may eliminate some strategic equilibria (NE)

Drawbacks of non-strategic equilibria

* Need to assume that it is ***common knowledge*** that all players are ***rational***
* The process often produces a very ***imprecise prediction*** about the outcome of the game

Games without a DE

* Most games ***have no*** dominant strategies
* Players ***cannot*** affect the outcome independently
* Without a dominant strategy the optimal decision of every player will ***depend*** on what the ***others*** do

  that is, each player is now ***concerned*** about the ***actions of other*** players

The Nash Equilibrium

- A ***more general*** but also ***weaker*** equilibrium concept is the ***Nash Equilibrium***

  A combination of strategies from which no player has an incentive to deviate unilaterally

- At the NE each player is doing the ***best*** they can ***given*** their expectations on their opponents' actions
- At the NE, any player who will ***deviate alone*** will end up ***worse off***

  the NE is a ***quasi-stable*** notion of equilibrium

*Equilibrium concepts

- Dominant Strategy

  "I'm doing the best I can no matter what you do –You're doing the best you can no matter what I do"

- Nash Equilibrium

  "I'm doing the best I can given what you are doing – You're doing the best you can given what I am doing"

- DE is a ***special case*** of the NE

Derivation of NE

P1 \ P2	Left	Middle	Right
Up	0 , 5	4 , 0	1 , 3
Center	4 , 4	0 , 5	1 , 3
Down	3 , 1	3 , 1	2 , 2

- Check for ***dominated strategies***
- Check each cell for ***unilateral deviation tendencies***
- The ***only cell*** that no one wants to deviate alone is (D,R)
- The NE does not have to be the ***social optimum***
  - If players ***decided together*** they would select (C,L)
  - But from (C,L) both players want to ***deviate unilaterally***

The sidewalk "game"

P1 \ P2	LHS	RHS
LHS	1 , 1	0 , 0
RHS	0 , 0	1 , 1

- There might be ***more than one*** NE
- Which one is the ***outcome*** of the game?
- Depends on
  - Where the game ***begins*** from, or
  - How ***initial perceptions*** are formed

Rationality and predictability

- The process that yields NE requires ***rationality***
- Does this mean that players can ***forecast*** the moves of their opponents?
- If there is a ***unique obvious way*** to play the game, everyone will understand this

  that is, if there is ***only one reasonable outcome*** for a game, this ought to be NE

What is the outcome in this game?

Oleg \ Tatiana	Left	Right
Up	0, 0	1, 1
Down	1B, 1B	0, 0

- *(D,L)* is a ***focal point***

Mixed strategies

* Some games ***do not have*** equilibria in pure strategies

		P2 C	P2 D
P1	A	0 , 1	1 , 0
P1	B	1 , 0	0 , 1

* We can select the appropriate ***mixture*** of strategies that maximizes the expected payoff

  assigning a ***probability*** to each non dominated action

* Each player can select a ***50-50 mixture*** of both his strategies and maximize his expected payoff

Mixing

		P2 C	P2 D	
P1	A	0 , 1	1 , 0	p
P1	B	1 , 0	0 , 2	$1-p$
		q	$1-q$	

* For P1:

$$E\Pi_A = 0\cdot q + 1\cdot(1-q) = 1-q$$
$$E\Pi_B = 1\cdot q + 0\cdot(1-q) = q$$

* Solving $E\Pi_A = E\Pi_B$

$$1-q=q \quad \Rightarrow \quad q=\frac{1}{2}$$

Mixing

		P2 C	P2 D	
P1	A	0 , 1	1 , 0	p
P1	B	1 , 0	0 , 2	$1-p$
		q	$1-q$	

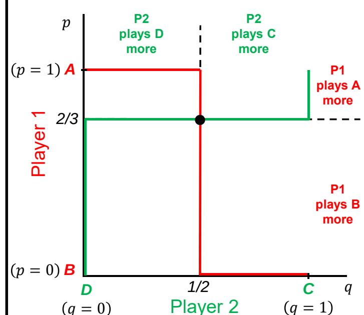
* For P2:

$$E\Pi_C = 1\cdot p + 0\cdot(1-p) = p$$
$$E\Pi_D = 0\cdot p + 2\cdot(1-p) = 2-2p$$

* Solving $E\Pi_C = E\Pi_D$

$$p = 2-2p \Rightarrow p=\frac{2}{3}$$

Mixed strategies – graph



		P2 C	P2 D
P1	A	0 , 1	1 , 0
P1	B	1 , 0	0 , 2

Existence of NE

* Nash Theorem:

  For an n-player game with a finite set of strategies for every player there exists at least one NE, possibly involving mixed strategies

Incompleteness

		P2 Left	P2 Middle
P1	Up	3 , ?	0 , ?
P1	Center	0 , ?	3 , ?
P1	Down	1 , ?	1 , ?

* How would you handle the above ***incomplete*** game?

Thank you!

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