Kosmas Marinakis, Ph.D.

Lecture 9

The Bertrand paradox

Industrial Economics

Competition in prices

- Let us now assume that the firms' ***choice variable*** is ***price*** instead of quantity
- To the seller, p and q have an ***one-to-one relationship*** through the demand function
- However, the two are ***fundamentally different*** at a ***strategic level***:
  - Increasing quantity unilaterally, increases market-share and thus ***increases profits***
  - Increasing price unilaterally, decreases market-share and thus ***decreases profits***

Bertrand (1883)

- Firm 1 and firm 2 face ***market demand*** $q(p)$
- ***Cost*** per unit is c
- ***Demand*** for each ***firm,*** $i = 1,2$ with $i \neq j$ is

$$q_i(p_i, p_j) = \begin{cases} q(p_i) & if \quad p_i < p_j \\ 0.5 \cdot q(p_i) & if \quad p_i = p_j \\ 0 & if \quad p_i > p_j \end{cases}$$

- Firms ***simultaneously*** announce ***binding*** prices p_i, p_j

Incentive for deviation

- For any $p_i > c$, firm j has an ***incentive*** to undercut

  grabs i's entire market share this way
- For any $p_i < c$, firm i has an ***incentive*** to raise p_i

  i bleeds money if prices below cost
- For $p_i = c$, firm j's best response is to ***match***

  neither i nor j have an incentive to deviate from this outcome
- ***Marginal cost pricing*** is the ***NE*** in Bertrand!
- This leads to ***zero profit***

$$\Pi_i = (p - c) \cdot 0.5 \cdot q_i\,(p) = 0$$

Bertrand equilibrium

- The zero-profit outcome in Bertrand is ***paradoxical***

  firms appear to not have any market power
- Notice the ***importance*** of the ***strategic variable***
  - In Cournot a firm believes that its rivals will sell a ***fixed*** quantity
  - In Bertrand a ***slight difference*** in price may change the market shares ***dramatically***
- In Bertrand, ***firms' demand curves*** are ***more elastic*** than under Cournot

  as a result, the Bertrand equilibrium is ***more efficient***, (greater output, lower prices)

Cournot or Bertrand?

- ***Bertrand*** is more descriptive of actual firm behavior – empirical evidence is more in accord with ***Cournot***
  - Cournot is appropriate when firms are ***capacity constrained*** and investments in capacity are ***sluggish***
  - Bertrand is more appropriate in situations where there are ***constant returns*** and firms can ***easily adjust quantity***
- We must consider the ***ex-ante characteristics*** of the industry
- But also ***test*** the models ex post for applicability

  whether the model's predictions are ***verified or falsified*** by actual industry behavior

The Kreps and Scheinkman (1983) hybrid

- A **two-stage** game
  - Firms first invest in ***capacity*** and then compete over ***prices***
  - Investment in capacity ***takes time*** and cannot be changed quickly - prices can be adjusted easily and rapidly
- At the ***equilibrium***
  - Each firm investing in capacity equal to its Cournot quantity
  - Then, the NE in prices, given capacity, has the firms pricing such that they produce to capacity
- The lower the initial investment in capacity the stronger the ***incentive to price more aggressively*** in the second stage

The Bertrand paradox

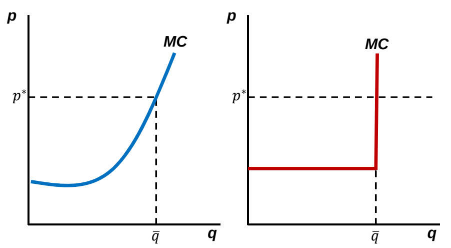
- Under ***Cournot*** firms enjoy positive profit
- Under ***Betrand*** both firms earn ***zero profit***

$$p = c, \qquad \Pi_i^* = \Pi_j^* = 0$$

  this solution is ***paradoxical***
- The ***source*** of the paradox is that a ***slight difference*** in price may change the market shares ***dramatically***
- This may ***not*** be the case under:
  1. Capacity constraints
  2. Differentiated products
  3. Repeated interaction

Capacity constraints

- Capacity constraints might be reflected from the ***demand*** in the market and/or the ***cost structure*** of the firm



Capacity constraints

Bertrand with capacity constraints

- Two firms with demand: $p = a - q_1 - q_2$
- For every firm, $i = 1{,}2$: $c_i = \begin{cases} 0 & , \quad q_i \le k \\ +\infty & , \quad q_i > k \end{cases}$
- ***Assume*** that both firms produce $q_i = k$, so that $p^* = a - 2k$ and $\Pi^* = (a - 2k)k$
- At capacity, clearly, there is no incentive to <u>cut the price</u>

  the firm is already sold out!
- What if firm i <u>increases the price</u> ***unilaterally***?

  firm j is still charging $p^* = a - 2k$ while producing $q_j = k$

Capacity constraints

Unilateral increase in p_i

- If p_i is <u>unilaterally</u> varying the demand can be ***inversed***

  q_j is fixed and firm i serves ***residual demand*** curve

$$p_i = (a - k) - q_i$$

- Profit for i is $\Pi_i = (a - k - q_i)q_i = (a - k)q_i - q_i^2$

  subject to $0 \le q_i \le k$
- Maximizing profit yields $d\Pi_i/dq_i = a - k - 2q_i$
- $\forall\, k \in [0, a/3]$ the FOC yields a ***bang-bang outcome,*** where the max Π_i is at the bound $q_i = k$
- This implies that for $k \in [0, a/3]$ ***there exists*** a NE so that

$$p_1 = p_2 = a - 2k > 0 = c$$

Capacity constraints

Rationing

- The preceding analysis is ***not as square*** as it appears
- If a firm is cheaper it serves at capacity and the more expensive one serves the ***residual*** demand

  who is buying from the cheaper firm?
- Rationing matters:
  - Four ***consumers*** with reservations 10, 6, 4, 1
  - Two ***firms***, with $c = 2$ and ***capacity*** 2 consumers each
  - The “cheap” firm ***charges*** $p_1 = 4$
  - ***If*** firm 1 serves the ’10’ and ‘6’ customers, $p_2 = 4$
  - ***If*** firm 1 serves the ‘6’ and ‘4’ customers, $p_2 = 10$

Rationing rules assumed

1. ***Proportional*** rationing (aka randomized)

   all consumers have the ***same probability*** to be rationed once they are priced-in

2. ***Efficient*** rationing

   high ***reservation*** consumers have higher incentives to put more effort into seeking the cheaper firm as they may acquire ***higher surplus*** from the transaction

3. Rationing by ***queuing***

   correlation between ***valuation of good*** and ***valuation of time*** will determine who will buy from the cheaper seller

* ***Arbitrage*** may be an issue with all methods

Differentiated products

* Another way to ***resolve the Bertrand paradox*** is by lifting the assumption for product homogeneity

  in this case, market shares are determined not just by prices, but by ***differences*** in design, performance, or durability of each firm's product

* In markets of differentiated goods it ***makes sense*** to compete using price instead of quantity

  customers ***will not desert*** the firm over a small price increase relative to the competition

Differentiation model

* Firms face symmetric linear ***demand*** curves

$$q_1 = a - dp_1 + p_2$$
$$q_2 = a - dp_2 + p_1$$

  quantity for each firm decreases with ***own-price*** but increases with ***cross-price***

* ***Marginal cost*** for both firms is c
* Firms choose prices ***simultaneously***

  decision on price is ***binding*** for the firm – cannot take it back

Reaction functions

* ***Profit*** for the two firms is

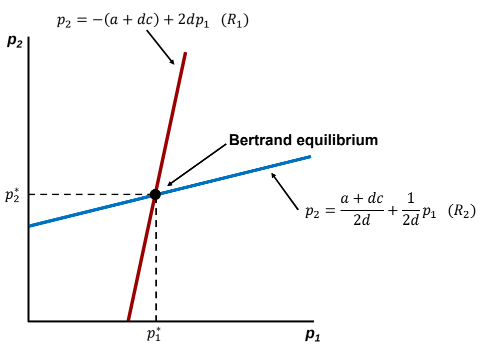
$$\Pi_1 = (a - dp_1 + p_2) \cdot p_1 - c \cdot (a - dp_1 + p_2)$$
$$\Pi_2 = (a - dp_2 + p_1) \cdot p_2 - c \cdot (a - dp_2 + p_1)$$

* Each firm i will ***maximize profit*** as $\partial \Pi_i / \partial p_i = 0$, which yields the ***reaction curves*** for each firm

$$p_2 = -(a + dc) + 2dp_1 \quad (R_1)$$
$$p_2 = \frac{a + dc}{2d} + \frac{1}{2d} p_1 \quad (R_2)$$

* For the reaction functions we ***can say*** that
  * They are both ***positively sloped***
  * They will ***intersect*** at the positive quartile, where $p_i^* > c$

Equilibrium



Differentiation application

* Firms face ***symmetric demand*** curves

$$q_1 = 10 - 2p_1 + p_2$$
$$q_2 = 10 - 2p_2 + p_1$$

* Marginal ***cost*** for both firms is $c = 1$
* ***Reaction functions*** become

$$p_2 = -12 + 4p_1 \quad (R_1) \quad , \quad p_2 = 3 + \frac{1}{4} p_1 \quad (R_2)$$

* ***Equilibrium*** will be

$$p_1^* = p_2^* = 4 \quad , \quad q_1^* = q_2^* = 6 \quad , \quad \Pi_1^* = \Pi_2^* = 18$$

Collusion benchmark

- Firms ***collude*** setting one common price $p_1 = p_2 = p$
- The two firm demand curves

$$q_1 = 10 - 2p_1 + p_2 \quad \text{and} \quad q_2 = 10 - 2p_2 + p_1$$

collapse into one demand curve

$$q = 20 - 2p \quad \text{or} \quad p = 10 - 0.5q$$

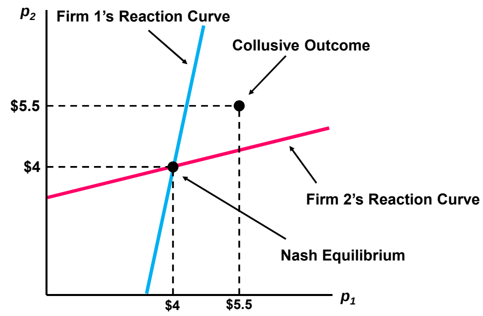
- With $c = 1$, ***maximization*** of profit yields

$$q_1^* = q_2^* = 4.5, \qquad p_1^* = p_2^* = 5.5$$

$$\Pi_1^* = \Pi_2^* = 20.25$$

- ***Firms benefit*** if they collude

Bertrand & collusion – application graph



The prisoners' dilemma (revisited)

girl \ boy	Confess	Deny
Confess	-5, -5	-1, -10
Deny	-10, -1	-2, -2

- The prisoner's dilemma is ***more than*** a game
- It is a ***philosophical device*** that makes the point that ***self-interest*** may lead to a ***social sub-optimal***

  in the game, acting towards the social best ***exposes*** the player to the self-interest of the others

- This analysis depicts with clarity the usual strategic competition environment in ***real business***

Thank you!

Kosmas Marinakis
www.kmarinakis.org
kmarinakis@hse.ru

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